ЛД-21 ИН

State budgetary educational institution of higher professional education
"NORTH-OSSETIAN STATE MEDICAL ACADEMY"
Ministry of Health Russian Federation

Department of the Chemistry and Physics

A GUIDE TO PRACTICAL AND LABORATORY STUDIES IN THE DISCIPLINE "PHYSICS, MATHEMATICS"

For students 1 course of the 31.05.01 General Medicine

Vladikavkaz, 2022

Developers:

Teacher of the Department
of Chemistry and Physics,
Gazdanova R.Y.

Teacher of the Department
of Chemistry and Physics,
Associate Professor Botsieva N. I.

Reviewers:

Magkoev T.T., Head of the Department of Condensed State Physics of the NOSU, doctor of physical and mathematical sciences, Professor

Babenko A.V., Teacher of the Department of Chemistry and Physics, Associate Professor

The manual was developed in accordance with the requirements of the Federal State Educational Standards of Higher Education in the areas of preparation 31.05.01 "General Medicine" and contains teaching materials for laboratory studies in the discipline "Physics, Mathematics".

For students of medical faculty

CONTENTS:

Theme: "BASIC CONCEPTS OF MATHEMATICAL ANALYSIS"

1. Scientific and methodological substantiation of the topic:

The concepts of derivative and differential are some of the basic concepts of mathematical analysis. The calculation of derivatives is necessary for solving many problems in physics and mathematics (finding the speed, acceleration, pressure, etc.). The importance of the concept of a derivative, in particular, is determined by the fact that the derivative of a function characterizes the rate of change of this function as its argument changes.

The methods for finding the derivatives and differentials of functions and their application form the basic problem of differential calculus. The necessity of the concept of derivative arises in connection with the formulation of the problem of calculating the speed of motion and finding the angle of the tangent to the curve. The inverse problem is also possible: determine the distance traveled by speed, and find the corresponding function from the tangent of the tangent angle. Such an inverse problem leads to the concept of an indefinite integral.

The notion of a definite integral is used in a number of practical problems, in particular, in problems of calculating the areas of plane figures, calculating the work done by the variable force, finding the mean value of the function.

In the mathematical description of various physical, chemical, biological processes and phenomena, equations containing not only the quantities studied but also their derivatives of various orders from these quantities are often used. For example, according to the simplest version of the bacterial propagation law, the rate of reproduction is proportional to the number of bacteria at a given time.

2. Theory:

1. Definition of a derivative.

Consider a function $y = f(x)$ at two points: x_0 and $x_0 + \Delta x$: $f(x_0)$ and $f(x_0 + \Delta x)$.
Here Δx means some small change of an argument, called an ***argument increment***; correspondingly a difference between the two values of a function: $f(x_0 + \Delta x) - f(x_0)$ is called a ***function increment***.

Derivative of a function $y = f(x)$ at a point x_0 is the limit:

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = y'$$

The operation of finding the derivative of a function f(x) is called ***differentiation*** of the function.

If this limit exists, then a function f (x) is a differentiable function at a point x_0.

Mechanical meaning of derivative. Consider the simplest case: a movement of a material point along a coordinate line, moreover, the motion law is given, i.e. a coordinate x of this moving point is the known function $x(t)$ of time t. During the time interval from t_0 till $t_0 + \Delta t$ the point displacement is equal to: $x(t_0 + \Delta t) - x(t_0) = \Delta x$, and its *average velocity* is: $v_a = \Delta x/\Delta t$. As $\Delta t \to 0$, then an average velocity value approaches the certain value, which is called an *instantaneous velocity* $v(t_0)$ of a material point in the moment t_0. But according to the derivative definition we have:

$$\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \to 0} \frac{x(t_0 + \Delta t) - x(t_0)}{\Delta t} = x'(t_0)$$

hence, $v(t_0) = x'(t_0)$, i.e. *a derivative of a coordinate with respect to time is a velocity.* This is a *mechanical meaning of a derivative.* Analogously to this, an *acceleration is a derivative of a velocity with respect to time*: $a = v'(t)$.

Loosely speaking, a derivative can be thought of as how much one quantity is changing in response to changes in some other quantity; for example, the derivative of the position of a moving object with respect to time is the object's **instantaneous velocity**.

The process of finding a derivative is called **differentiation**. The reverse process is called **antidifferentiation**.

Formulas of differentiation.

1) The derivative of any constant *(C)′=0*.
2) The derivative of the product of a constant by a function *(C**u**)′=C**u**′*.
3) The derivative of the sum of finite number of differentiable functions is equal to the corresponding sum of the derivatives of these functions: ***(u+v-w)′=u′+v′-w′***.
4) The derivative of a product of two differentiable functions: ***(uv)′=u′v+uv′***.
5) The derivative of a fraction (the quotient of two functions): ***(u/v)′=(u′v-uv′)/v²***.
6) The derivative of a composite function ***y=f(u)***, where ***u=g(x)*** (chain rule):

$$y'_x = y'_u \cdot u'_x.$$

The derivative of a composite function is equal to the product of the derivative of the given function with respect to the intermediate argument by the derivative of the intermediate argument with respect to x.

Derivatives of elementary functions

Kind of function	Function	Derivative
Power function	x^α	$\alpha x^{\alpha-1}$
	$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
	$\frac{1}{x}$	$-\frac{1}{x^2}$
Exponential function	a^x	$a^x \ln a$
	e^x	e^x
Logarithmic function	$\log_a x$	$\frac{1}{x\ln a}$
	$\ln x$	$\frac{1}{x}$
Trigonometric functions	$\sin x$	$\cos x$
	$\cos x$	$-\sin x$
	$\tan x$	$\frac{1}{\cos^2 x}$
	$\cot x$	$-\frac{1}{\sin^2 x}$
Inverse trigonometric functions	$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
	$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
	$\tan^{-1} x$	$\frac{1}{1+x^2}$
	$\cot^{-1} x$	$-\frac{1}{1+x^2}$

2. Differential of function

The principal part of the increment of the function being linear in Δx is called the ***differential of the function*** and is denoted as ***dy***.

Differential of a function is a product of a derivative $f'(x_0)$ and an increment of argument Δx:

$$df = f'(x_0) \cdot \Delta x.$$

If $y=x$ then $y_x'=1$ and, consequently, $dx=\Delta x$. The differential of the independent variable x coincides with its increment Δx.

New formula for the differential of the function:

$$dy = y' \cdot dx$$

Properties of differential

1. $d(C) = 0, C - const$
2. $d(u \pm v) = du \pm dv$
3. $d(u \cdot v) = vdu + udv$
4. $d\left(\frac{u}{v}\right) = \frac{vdu - udv}{v^2}$

3. Concepts of primitive and indefinite integral.

In many problems it is necessary to find a new function whose derivative is known.

Primitive. A continuous function $F(x)$ is called a *primitive* for a function $f(x)$ on a segment X, if for each $x \in X$

$$F'(x) = f(x).$$

E x a m p l e . The function $F(x) = x^3$ is a primitive for the function $f(x) = 3x^2$ on the interval $(-\infty; +\infty)$, because $F'(x) = (x^3)' = 3x^2 = f(x)$ for all $x \in (-\infty; +\infty)$. It is easy to check, that the function $x^3 + 13$ has the same derivative $3x$, so it is also a primitive for the function $3x^2$ for all $x \in (-\infty; +\infty)$.

It is clear, that instead of 13 we can use <u>any</u> constant.

Thus, *the problem of finding a primitive has an infinite set of solutions.*

This fact is reflected in the definition of an *indefinite integral:*

Indefinite integral of a function $f(x)$ on a segment X is a *set of all its primitives*. This is written as :

$$\int f(x)dx = F(x) + C,$$

and read "the indefinite integral of f(x) with respect to dx".

The function **f(x)** is called the ***integrand***, the expression **f(x)dx** is the ***element of integration***, C – ***integration constant***.

Geometrical meaning of the indefinite integral is given by Fig.1.

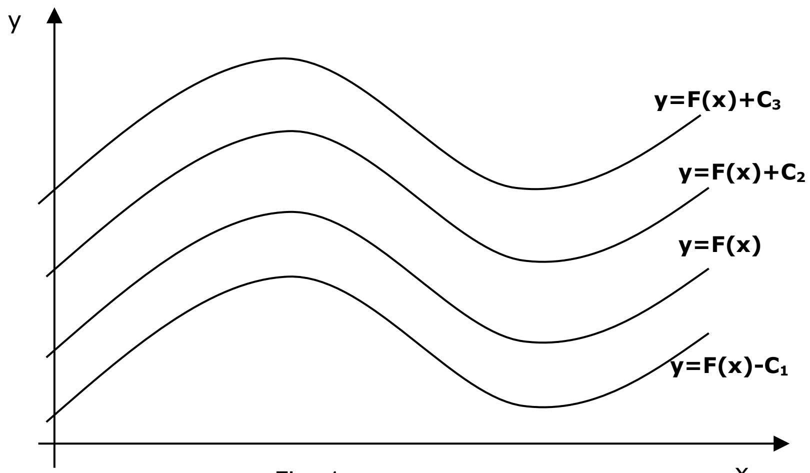


Fig. 1

Differentiation and Integration constitute the two fundamental operations in single-variable calculus.

Properties of indefinite integral.

1^0 If a function $f(x)$ has a primitive on a segment X, then for interior points of this segment:

$$\left(\int f(x)dx\right)' = f(x).$$

2^0 If $f(x)$ is a continuous function on a segment X and differentiable in interior points of this segment, then:

$$d\left(\int f(x)dx\right) = f(x)dx.$$

3^0 If $f(x)$ is a continuous function on a segment X and differentiable in interior points of this segment, then:

$$\int dF(x) = F(x) + C.$$

4^0 If a function $f(x)$ has a primitive on a segment X, and k – a number, then:

$$\int\left(\sum_{i=1}^{n} k_i f_i(x)\right)dx = \sum_{i=1}^{n}(k_i \int f_i(x)dx).$$

Table of integrals

Using the table of derivatives, we compile a table of indefinite integrals:

1. $\int dx = x + C$
2. $\int x^n dx = \dfrac{x^{n+1}}{n+1} + C \qquad n \neq -1$
3. $\int \dfrac{dx}{x} = \ln|x| + C$
4. $\int a^x dx = \dfrac{a^x}{\ln a} + C$
5. $\int e^x dx = e^x + C$
6. $\int \sin x dx = -\cos x + C$
7. $\int \cos x dx = \sin x + C$
8. $\int \dfrac{dx}{\cos^2 x} = \tan x + C$
9. $\int \dfrac{dx}{\sin^2 x} = -\cot x + C$

10. $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

11. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$

Methods of integration.

1) Direct method.

An integrand is reduced by the elementary simplification to the table forms.

For example:

$$\int\left(3x^5 - \cos x + \frac{1}{x}\right)dx = 3\int x^5 dx - \int \cos x dx + \int \frac{1}{x} dx =$$

$$= 3\frac{x^6}{6} - \sin x + \ln x + C = \frac{x^6}{2} - \sin x + \ln x + C$$

2) Integration by substitution (exchange).

*If a function f (z) is given and has a primitive at z∈**Z** , a function z=φ(x) has a continuous derivative at x∈**X** , and φ(**X**)⊂**Z** , then the function F(x) = f [φ(x)]•φ'(x) has a primitive in **X** and:*

$\int F(x)dx = \int f[\varphi(x)]\cdot \varphi(x)dx = \int f(z)dz$.

For example: Find the integral $\int e^{\cos x} \sin x dx$

To get rid of the trigonometric function in power we assume *cosx* = *t*, then *dt* = *-sinxdx*. Then exchanging, we have

$$\int e^{\cos x} \sin x dx = \begin{bmatrix} t = \cos x \\ dt = (\cos x)' dx = -\sin x dx \end{bmatrix} =$$

$$= \int e^t (-dt) = -\int e^t dt = -e^t + C = -e^{\cos x} + C.$$

3) Method of integration by parts.

This method is used in case of the integrand as a product of two functions.

The formula of integration by parts is a consequence of the formula for the differential of the product of two functions: $d(uv) = udv + vdu.$ Hence $udv = d(uv) - vdu$ and $\int udv = \int d(uv) - \int vdu.$. Finally, we get the formula:

$$\int udv = uv - \int vdu. \qquad (*)$$

For example: Find $\int x \cos x dx$.

Applying the formula (*), we get $\int x\cos x dx = \begin{bmatrix} x = u & du = dx \\ \cos x dx = dv & v = \int \cos x dx = \sin x \end{bmatrix} =$

$= x\sin x - \int \sin x dx = x \sin x + \cos x + C$.

The formula (*) can be used several times. The most important cases of using the method of integration by parts are given in the next table.

Element of integration **Udv**	Designation as a **u**	Designation as a **dv**	How many times?
$P_n(x)e^x dx$	$P_n(x)$	$e^x dx$	N
$P_n(x)\log_a x dx$	$\log_a x$	$P_n(x)dx$	1
$P_n(x)\cos x dx$	$P_n(x)$	$\cos x dx$	N
$P_n(x)\sin x dx$	$P_n(x)$	$\sin x dx$	N
$P_n(x)\tan^{-1} x dx$	$\tan^{-1} x$	$P_n(x)dx$	1
$e^x \cos x dx$	e^x $\cos x$	$\cos x dx$ $e^x dx$	2
$e^x \sin x dx$	e^x $\sin x$	$\sin x dx$ $e^x dx$	2

4. Concept of definite integral.

Consider a continuous function $y = f(x)$, given on a segment $[a, b]$ and saving its sign on this segment (Fig.2).

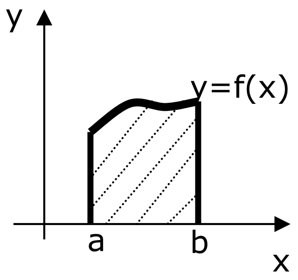


Fig. 2

The figure, bounded by a graph of this function, a segment $[a, b]$ and straight lines $x = a$ and $x = b$, is called a ***curvilinear trapezoid***. Consider the way to *calculate an area of a curvilinear trapezoid*.

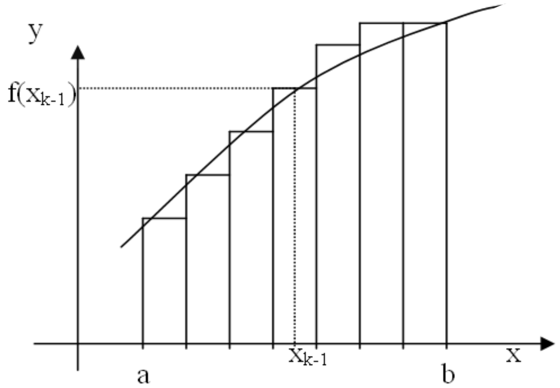


Fig. 3

Divide a segment $[a, b]$ into n segments of an equal length by points:
$x_0 = a < x_1 < x_2 < x_3 < \ldots < x_{n-1} < x_n = b$
and let $\Delta x = (b - a) / n = x_k - x_{k-1}$, where $k = 1, 2, \ldots, n-1, n$.
In each of segments $[x_{k-1}, x_k]$ as on a base we'll build a rectangle of height $f(x_{k-1})$.
An area of this rectangle is equal to:

$$f(x_{k-1}) \cdot \Delta x = \frac{b-a}{n} f(x_{k-1})$$

And a sum of areas of these rectangles is equals to:

$$S_n = \frac{b-a}{n}\left(f(x_0) + f(x_1) + \ldots + f(x_{n-1})\right)$$

In view of continuity of a function $f(x)$ a union of the built rectangles at great n (i.e. at small Δx) "almost coincides" with our curvilinear trapezoid. Therefore, $S_n \approx S$ at great values of n. It means, that $S_n \to S$ at $n \to \infty$. This limit is called *an integral of a function* $f(x)$ *from* a *to* b or a ***definite integral***:

$$\int_a^b f(x)dx, i.e. \mathrm{S_n} \to \int_a^b f(x)dx \text{ at } \mathrm{n} \to \infty$$

Numbers a and b are called ***limits of integration***, $f(x)\,dx$ – an ***integrand***.
So, if $f(x) \geq 0$ on a segment $[a, b]$, then an area S of the corresponding curvilinear trapezoid is represented by the formula:

$$\lim_{\Delta \to 0} S_n = \int_a^b f(x)dx$$

From the definition of definite integral the geometric meaning follows:
numerically the definite integral is equal to the area of curvilinear trapezoid.

Newton – Leibniz formula.

If $F(x)$ is primitive for the function $f(x)$ on a segment $[a, b]$, then

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$$

Properties of definite integrals.

1^0. Invariance of the variable of integration.

$$\int_a^b f(x)dx = \int_a^b f(t)dt = \int_a^b f(u)du = ...$$

2^0. Interchange of limits.

$\int_a^b f(x)dx = -\int_b^a f(x)dx$. Particularly, $\int_a^a f(x)dx = 0$.

3^0. Additivity of the definite integral. (this property follows from additivity of the area.)

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

4^0. Linearity of the definite integral.

$$\int_a^b \left(\sum_{i=1}^n k_i f_i(x) \right) dx = \sum_{i=1}^n \left(k_i \int_a^b f_i(x)dx \right).$$

5^0. Connection between the sign of the integral and the sign of the integrand.

If $sign f(x) = const$ on *[a,b]*, then $sign \int_a^b f(x)dx = sign f(x)$ on *[a,b]*.

I.e. if $f(x) \geq 0$ on [a,b], then $\int_a^b f(x)dx \geq 0$ too.

6^0. Average value of the function on [a,b].

$$\int_a^b f(x)dx = \mu(b-a).$$

The value μ is called the average of function f(x) on [a,b].

Methods of integration.

1) *Direct method* (*according to the Newton-Leibniz formula*)

For example: $\int_{-2}^{-1}\frac{dx}{x}=\ln|x|\Big|_{-2}^{-1}=\ln|-1|-\ln|-2|=\ln 1-\ln 2=-\log 2$.

2) ***Changing a variable*** (in contrast to the indefinite integration it is not necessary to return to the previous argument, only the new limits should be found).

For example:

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} e^{\sin x}\cos x dx=\begin{bmatrix}\sin x=t \\ \\ dt=\cos xdx;\end{bmatrix} \text{ find the new limits } \left[\begin{array}{c|c|c} \frac{x}{t} & \frac{\frac{\pi}{3}}{\frac{\sqrt{3}}{2}} & \frac{\frac{\pi}{2}}{1}\end{array}\right]=\int_{\frac{\sqrt{3}}{2}}^{1} e^t dt=e^t\Big|_{\frac{\sqrt{3}}{2}}^{1}=e-e^{\frac{\sqrt{3}}{2}}.$$

3) ***Integration by parts***:

$$\int_a^b udv=uv\Big|_a^b-\int_a^b udv$$

For example:

$$\int_0^{\frac{\pi}{2}} x\cos xdx=\begin{bmatrix}x=u & du=dx \\ \cos xdx=dv & v=\sin x\end{bmatrix}=x\sin x\Big|_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\sin xdx=$$

$$=(x\sin x+\cos x)\Big|_0^{\frac{\pi}{2}}=(\frac{\pi}{2}\sin\frac{\pi}{2}+\cos\frac{\pi}{2})-(0\sin 0+\cos 1)=\frac{\pi}{2}-1.$$

5. Elementary differential equations.

Main concepts.

A **differential equation** is an equation involving an unknown function and its derivatives

There are two types of differential equations:

- ordinary differential equations, involving only one independent variable;
- partial differential equations, which involve more then one independent variables.

We'll concern ourselves with only one variable.

Differential equations are classified according to the highest derivative which occurs in them.

Generally, differential equations of n-th order are written in form:

$$\mathbf{F(x, y, y', y'', \ldots, y^{(n-1)}, y^{(n)})=0.}$$

The **order** of the differential equation is the order of the highest derivative of the unknown function involved in the equation

A **linear differential equation** of order *n* is a differential equation written in the following form:

$$a_n(x)\frac{d^n y}{dx^n}+a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}}+\ldots+a_1(x)\frac{dy}{dx}+a_0(x)y=f(x),$$

where $a_n(x)$ is not the zero function.

Note that some may use the notation ***y, y', y'', y''', y^{IV},…*** for the derivatives**.**

To solve a differential equation it is necessary to find all unknown functions which at substituting covert the equation into an identity.

Any differential equation has two solutions: *general* and *particular*.

A problem in which we are looking for the unknown function of a differential equation where the values of the unknown function and its derivatives at some point are known is called an **initial value problem** (in short IVP).

If no initial conditions are given, we call the description of all solutions to the differential equation the **general solution**

The general solution of a differential equation of n-th order contains a number of arbitrary constants which is equal to the order of the equation:

$$y=f(x, C_1, C_2, \dots, C_{n-1}, C_n)$$

Very often general solution is in an implicit form (general integral):

$$F(x, y, C_1, C_2, \dots, C_n)=0.$$

Particular solution is a result of Cauchy problem:

$$\begin{cases} F(x,y,y',y'',\dots,y^{(n)})=0 \\ \left.\begin{array}{l} y(x_0)=y_0 \\ y'(x_0)=y_1 \\ \dots\dots\dots\dots \\ y^{(n-1)}(x_0)=y_{n-1} \end{array}\right\} \end{cases}$$

First order linear equations

A ***first order linear differential equation*** has the following form:

$$\frac{dy}{dx}+p(x)y=q(x)$$

The general solution is given by

$$y=\frac{\int u(x)q(x)dx+C}{u(x)}$$

where

$u(x)=\exp\left(\int p(x)dx\right)$

called the **integrating factor**. If an initial condition is given, use it to find the constant *C*.

Here are some practical steps to follow:

1.

If the differential equation is given as

$a(x)\frac{dy}{dx}+b(x)y=c(x)$

rewrite it in the form

$\frac{dy}{dx}+p(x)y=q(x)$

where

$p(x)=\frac{b(x)}{a(x)}\ and\ q(x)=\frac{c(x)}{a(x)}$

2.

Find the integrating factor

$u(x)=e^{\int p(x)dx}$

3.

Evaluate the integral $\int u(x)q(x)dx$

4.

Write down the general solution

$y=\frac{\int u(x)q(x)dx+C}{u(x)}$.

5. If you are given an IVP, use the initial condition to find the constant *C*.

Example: Find the particular solution of:

$y' + y\tan x = \cos^2 x,\ y(0) = 2$

Solution: Let us use the steps:

Step 1: There is no need for rewriting the differential equation. We have

$p(x) = \tan x$ and $q(x) = \cos^2 x$

Step 2: Integrating factor

$$u(x) = e^{\int \tan x dx} = e^{\int \frac{\sin x}{\cos x} dx} = e^{-\int \frac{d(\cos x)}{\cos x}} = e^{-\ln(\cos x)} = e^{\ln \frac{1}{\cos x}} = \frac{1}{\cos x}$$

Step 3: We have

$$\int \frac{1}{\cos x} \cos^2 x dx = \int \cos x dx = \sin x$$

Step 4: The general solution is given by

$$y = \frac{\sin x + C}{\frac{1}{\cos x}} = (\sin x + C)\cos x$$

Step 5: In order to find the particular solution to the given IVP, we use the initial condition to find *C*. Indeed, we have

$y(0) = 2$

$2 = (\sin 0 + C)\cos 0$

$C = 2$

Therefore the solution is

$y = (\sin x + 2)\cos x$.

Note that you may not have to do the last step if you are asked to find the general solution (not an IVP).

SEPARABLE EQUATIONS

If it is possible to rearrange the terms of the equation into two groups, each containing only one variable, the variables are said **to be separable**.

The examples of function with separable variables:

$e^{x+y} = e^x e^y$, $x^2y+y=y(x^2+1)$, $xy+2y-3x-6=(x+2)(y-3)$.

The differential equation of the form $\frac{dy}{dx} = f(x,y)$ is called **separable**, if $f(x,y) = h(x)\,g(y)$;

that is, $\frac{dy}{dx} = h(x)g(y)$

In order to solve it, perform the following steps:

The algorithm of solution.

1) Let us consider the equation $f_1(x) f_2(y) y' = \varphi_1(x)\varphi_2(y)$;

2) Change y' for $\frac{dy}{dx}$ and multiply by dx: $f_1(x) f_2(y) dy = \varphi_1(x)\varphi_2(y) dx$;

3) Divide the both parts of the equations by $f_1(x)\cdot\varphi_2(y)$: $\frac{f_2(y)}{\varphi_2(y)} dy = \frac{\varphi_1(x)}{f_1(x)} dx$;

4) ‘Integrate directly : $\int \frac{f_2(y)}{\varphi_2(y)} dy = \int \frac{\varphi_1(x)}{f_1(x)} dx$ or $F(y) = \Phi(x) + C$.

Example 1. Solve the differential equation $xy' + y = 0$.

Solution: To separate the variables divide throughout by **xy** the equation $xdy = -ydx$:

$$\frac{dy}{y} = -\frac{dx}{x}.$$

Then $\int \frac{dy}{y} = -\int \frac{dx}{x}$ and finally $\ln|y| = -\ln|x| + C_1$.

If the constant be written in the form C_1=lnC then $\ln|y| = \ln C - \ln|x|$ or $y = \frac{C}{x}$ - it is the general solution.

3. The purpose of the activity of students in the class:

The student should know:

1. Definitions of the derivative and differential of a function.
2. Physical and geometric meanings of the derivative.
3. The table of derivatives of the basic elementary functions.
4. Rules of differentiation.
5. Analytical and geometric meanings of the differential.
6. The concepts of indefinite and definite integrals.
7. The table of basic integrals.
8. Basic properties of indefinite and definite integrals.
9. Basic methods of integration.
10. Definition of an ordinary differential equation.
11. The concept of general and particular solutions of a differential equation.
12. Definition of a differential equation with separable variables and an algorithm for solving it.

The student should be able to:

1. Calculate the derivatives and differentials of functions.
2. Calculate the indefinite and definite integrals by different methods.
3. Calculate the mean values of functions, the area of plane figures, the work of variable force.
4. Find solutions of differential equations with separable variables.

4. Training content:

Theoretical part:

1. Problems leading to the concept of a derivative of a function.
2. Geometric and physical meanings of the derivative.
3.Products of complex functions.
4. Differential of a function. Geometric and analytical meanings of the differential.
5. Primitive functions. The indefinite integral. Basic properties of an indefinite integral.
6. Basic methods of integration.
7. Problems that lead to the notion of a definite integral.
8. The Newton-Leibniz formula. Basic properties of a definite integral.
9. Applications of a certain integral: calculating areas of plane figures, calculating the average values of functions, calculating the work of a variable force.
10. First-order differential equations with separable variables.

Practical part:

1. Find the derivatives and differentials of the functions:

1) y$= \frac{2}{\sqrt{x}} + \frac{\sqrt{x}}{2}$; 4) y= $\sqrt[3]{x^2} + \frac{1}{x} + 2$;

2) y=$\frac{x^3}{\sin^3 3x}$; 5) y=arccosx ;

3) $y=e^{3x+1}$; 6) $y=\sqrt{\sin 2x}$.

2. Find the integrals using the decomposition method:

1) $\int \frac{1-3x}{x^2}dx$; 3) $\int \frac{(1-x)^2}{x\sqrt{x}}dx$;

2) $\int \frac{1}{\sqrt{x}} + \cos 5x dx$; 4) $\int \left(3x^2 + \frac{4}{x}\right)dx$.

3. Find the integrals by the method of changing the variable:

1) $\int \frac{\cos x}{\sin 4x}dx$; 3) $\int \sin x \cos^4 x dx$;

2) $\int \frac{\cos x dx}{(1+\sin x)^2}$; 4) $\int \frac{e^x}{e^x+2}dx$.

4. Compute the definite integrals by the method of changing the variable:

1) $\int_0^9 \sqrt{5-2x}dx$ 3) $\int_0^{\frac{\pi}{3}} \tan x dx$

2) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^4 x}dx$ 4) $\int_0^1 \frac{e^x}{e^x+4}dx$

5. Calculate the areas of figures bounded by lines:

1) $y=x^2$ и $y=x^3$. 2) $y=\sqrt{x}$ и $y=x$.

6. Solve first-order differential equations with separable variables:

1) $y' = 2x^2+1$; 5) $y'(x+1)=1$;

2) $y'=5y$; 6) $e^y y'=1$;

3) $3xdx= 2ydy$; 7) $e^x y'=1$;

4) $x^2 dy - \frac{1}{2} y^2 dx = 0$; 8) $y' = 2x^2 + 1$.

5. List of questions to check the initial level of knowledge:

1. Define the derivative of a function.
2. Formulate the basic rules of differentiation.
3. Write down the formula for the derivative of a composite function.

4. What are the physical and geometric meanings of the derivative of a function?
5. What is the differential of a function?
6. What is the geometric meaning of the differential of a function?
7. Give the definition of the antiderivative function.
8. Write the basic properties of the indefinite integral.
9. Write down the integration formula by parts.
10. Give a geometric interpretation of a definite integral.
11. Write down the Newton-Leibniz formula
12. Give the definition of an ordinary differential equation.
13. What is the difference between the particular and general solutions of a differential equation?

6. List of questions for verifying the final level of knowledge:

1. What is the physical meaning of the second-order derivative?
2. What is the analytical meaning of the differential?
3. How is the differential used to calculate the errors?
4. What are the two main tasks related to the physical and geometric interpretation of the derivative, solved by integration?
5. How to verify the correctness of finding an indefinite integral?
6. Can the result of calculating a definite integral be verified by differentiation?
7. What is the basis for the application of a definite integral for calculating the areas of plane figures?
8. Do partial solutions of a differential equation contain arbitrary constants?
9. Create a sequence of solving a first-order differential equation with **separating variables.**

7. Chronocard of the lesson:

1. Organizational moment - 5 min.
2. Analysis of the topic - 30 min.
3. The solution of examples and tasks-60 min.
4. Current knowledge control -35 min.
5. Summing up the lessons - 5 min.

8. List of educational literature for the lesson:

1. L.V. Kukharenko, O.V. Nedzved, M.V. Goltsev, V.G. Leshchenko, “Medical and biological physics for medical students”, Minsk BSMU 2016.

Topic: "ELEMENTS OF THE PROBABILITY THEORY"

1. Scientific and methodological substantiation of the topic:

The theory of probability studies the regularities that manifest themselves in the study of such experiments, the concrete result of which before their implementation can not be predicted with certainty. So, with a single coin toss, you can not determine in advance, the coat of arms or the figure will drop out. At the same time, the results of numerous experiments show that the coat of arms and the figure are about the same amount. Thus, despite the random nature of the result of each experiment, there are some patterns for the results of many similar experiments.

Many random events can be quantified by random variables that take values depending on the coincidence of random circumstances.

In practical activities, a medical worker constantly deals with such quantities (the number of patients at the doctor's appointment, the patient's body temperature, blood pressure, dosage of the drug, etc.). Therefore, it is necessary to know how these quantities are obtained, and what is their accuracy. The mathematical basis of these questions are probability theory and mathematical statistics.

2. Theory:

1. Classical (theoretical) and statistical (empirical) probability definition.

In the real world events cannot be predicted with total certainty. The best one can do is to say how likely they are to happen, using the idea of probability.

Probability theory is the branch of mathematics deals with analysis of random events. Random event is event which at realization of a complex of conditions may occur or may not occur. Events can be named with capital letters: A, B, C… Examples of random events: the birth of girl in the family; the birth of a child with a predicted weight; the emergence of epidemic disease in the region in a certain period of time.

More generally, if there is a situation in which there are n equally likely outcomes, and the event A consists of exactly m < n of these outcomes, one can say that the classical probability P(A) of the event A is:

$$P(A)=\frac{m}{n} \qquad (1)$$

This definition can be applied in a situation in which all possible outcomes and the outcomes in the events can be counted.

Example: A box contains 3 red marbles, 1 blue marble, and 4 yellow marbles. One marble is drawn at random. There are now 8 equally likely marbles that can be drawn:

P (draw one of the eight marbles and it is red) = 3/8.

P (draw one of the eight marbles and it is blue) = 1/8.

P (draw one of the eight marbles and it is yellow) = 4/8.

So the classical probability definition is based on the physics of the experiment, but does not require the experiment to be performed. For example, we know that the probability of a balanced coin turning up heads is equal to 0.5 without ever performing trials of the experiment.

The probability of event accepts value between zero and unit: $0 \leq P(A) \leq 1$. If P(A) = 1, event A is certain event. A certain event is certain to occur. If P(A) = 0, it is impossible event. An impossible event has no chance of occurring. In other cases A is a random event and its probability $0 < P(A) < 1$.

Examples: The Christmas will be celebrated on the 25th of December this year. This is a certain event. When a number cube is rolled 7 is an impossible event. The sunny day in London is a random event.

Statistical probability. Since random experiments can be repeated as many times as we wish under identical conditions (in theory) we can measure the relative frequency of the occurrence of an event. Statistical (empirical) probability is based on long-run relative frequencies. The relative frequency f of event A in a given set of N trials is the ratio of the number M of those trials in which A occurs to the total number of trials N:

$$f = M/N.$$

Statistical probability of event is a limit to which relative frequency of event tends at unlimited increase of the general number of tests:

$$P(A) = \lim_{N \to \infty} \frac{M}{N}, \tag{2}$$

2. Types of random events

There are 3 main types of random events: disjoint events, independent events and dependent events.

1. Two events are ***disjoint*** if it is impossible for them to occur together.

Example: anyone cannot be both male and female, nor can they be aged 20 and 30.

Events M_1, M_2, …, M_k form the full group of events if at any tests there can be only one of them, and can't be any other events.

Example 1: Getting a student on one test mark «1», or «2», or «3», or «4», or «5», or «6», or «7», or «8», or «9» or «10» — the events are disjoint, since one of these marks exclude the other on the same exam. These events form a full group of events.

Example 2: Let P(A) is the probability of death for some diseases; it is known and is equal to 2 %. Then the probability of a successful outcome in this disease is 98 %. These events form full group of events.

2. Two or more events are ***independent*** if the occurrence of one of the events does not change the probability of the other events. That is, the events have no influence on each other. Two events A and B are independent if when one of them happens, it doesn't affect the other one happening or not.

Examples: choosing a marble from a jar and landing on heads after tossing a coin; choosing a 3 from a deck of cards, replacing it, and then choosing an ace as the second card.

If two events are independent then they cannot be disjoint.

3. Two or more events are ***dependent*** if the result of one event is affected by the result of other events. For dependent events A and B two types of probabilities are known: conditional probability and unconditional one.

The ***unconditional*** probability P(B) of an event B is the probability that event B will occur before an event A. The conditional probability P(B/A) of an event B, in relation to event A, is the probability that event B will occur given the knowledge that an event A has already occurred.

Example: taking out a marble from a bag containing some marbles and not replacing it, and then taking out a second marble are dependent events.

3. Probabilities addition and multiplication rules

Probability addition rule:

1. When two events, A and B, are disjoint, the probability that event A or event B will occur is the sum of the probabilities of each event:

$$P(A \text{ or } B) = P(A) + P(B) \tag{3}$$

2. For some disjoint events M1, M2, …, Mk:

$$P(M1 \text{ or } M2 \text{ or } \dots \text{ or } Mk) = P(M1) + P(M2) + \dots + P(Mk)$$

3. For full group of events:

$$\sum_{i=1}^{n} p_i = 1 \tag{4}$$

Probability multiplication rule for independent events:

1. If A and B are independent events, the probability of both events occurring is the product of the probabilities of the individual events:

$$P(A \text{ and } B) = P(A) \cdot P(B) \qquad (5)$$

Example: A drawer contains 3 red paperclips, 4 green paperclips, and 5 blue paperclips. One paperclip is taken from the drawer and then replaced. Another paperclip is taken from the drawer. What is the probability that the first paperclip is red and the second paperclip is blue?

Because the first paper clip is replaced, the sample space of 12 paperclips does not change from the first event to the second event. The events are independent.

P(red and blue)= P(red)· P(blue)= $\frac{3}{15} \cdot \frac{5}{15} = \frac{15}{144} = \frac{5}{48}$

2. For some disjoint events M1, M2, …, Mk:

P(M1 and M2 and ... and Mk) = P(M1) · P(M2) · ... · P(Mk).

Probability multiplication rule for dependent events: If A and B are dependent events, the probability joint appearance of two dependent events A and B is equal to product of the unconditional probability of first event by the conditional probability of another one:

$$P(A \text{ and } B) = P(A) \cdot P(B/A)\ (2.6) \qquad (6)$$

or

$$P(B \text{ and } A) = P(B) \cdot P(A/B). \qquad (6a)$$

In second case the first occurs event B and its probability is equal P(B) and for event A the conditional probability P (A/B) is realized.

Example: A drawer contains 3 red paperclips, 4 green paperclips, and 5 blue paperclips. One paperclip is taken from the drawer and is not replaced. Another paperclip is taken from the drawer. What is the probability that the first paperclip is red and the second paperclip is blue?

Because the first paper clip is not replaced, the sample space of the second event is changed. The sample space of the first event is 12 paperclips, but the sample space of the second event is now 11 paperclips. The events are dependent.

P(red and blue)= *P*(red)· *P*(blue/ red)= $\frac{3}{12} \cdot \frac{5}{11} = \frac{15}{132} = \frac{5}{44}$.

4. Bernoulli formula

Let's **n** independent repeated trails under the conditions:

1. the number of trails **n** is definite;
2. in every trail there are two outcomes (**A**-success, **Ā**-failure);
3. **P(A)=p** does not depend on the number of trail;
4. the probability $\mathbf{P_n(m)}$ that in **n** trails the event A has occurred **m** times sharply is define by the Bernoulli formula:

$$P_m(n) = \frac{n!}{m!(n-m)!} p^m q^{n-m} \quad (7)$$

5. Poisson formula

Let's find the probability that the event A has occurred sharply **m** *times at large number of trail (*$\boldsymbol{n}$*>>1) and small probability (P(A)<<1). In this case it is impossible to use neither Bernoulli formula.*

At large **n** and small **p** for calculation of required probability the Poisson formula is used:

$$P_n(m) \approx \frac{\mu^m}{m!} e^{-\mu}, , \qquad (8)$$

where $\mu = n \cdot p = const$

The Poisson formula is used widely in queuing theory, demography, social medicine.

6. Random variables and their characteristics

§1. Concept of random variable.

The term "a random variable" is the main concept of the probability theory.

A **random variable** is a variable whose possible outcomes in an experience are taken to be some value before hand unknown

Examples: Weight and height of human, number of student on the lecture, number of the practicles etc.

Random variables are these:

- **discrete** (whose possible outcomes are taken the to be numerical. For example: number of particles.)
- ***continuous*** (whose possible outcomes are taken the vale from finite or infinite interval.For example: Blood pressure, human weight, temperature, etc).

Random variables are designated by capital Latin letters X, Y, Z and possible values are – by ordinal (порядковый) letters x, y, z..

§2. Discrete random variables and their characteristics

2.1. Probability distibution (распределение) of discrete random variable

The $\{x_1, x_2, \ldots x_n\}$ is the set of outcomes of discrete random variable (DRV) X. $P(x_i) \equiv p_i$ – the probability of the possible value x_i.

The sum of the probabilities of all possible values of X is equal to 1 (as for total group of events):

$$\sum_{i=1}^{n} p_i = 1 \qquad (9)$$

This formula is called "the norming condition".

The discrete random variable are said to define if all possible values and their probabilities are given .

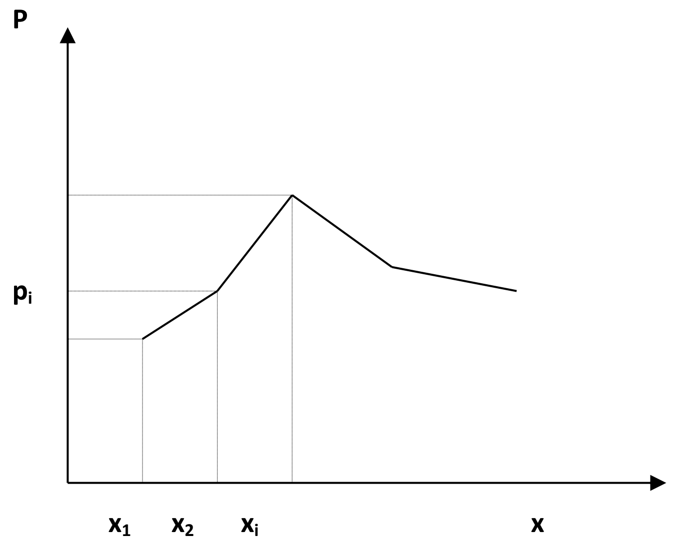
The probability distibution (or probability function) of a DRV is called the correspondence between the possible values and their probabilities .

The probability function of X will sometimes be given by means of either formula or a table of its values.

Table form of represantion of the probability function:

Value x	x_1	x_2	…..	x_i	…..	x_n
Probability p	p_1	p_2	…..	p_i	…..	p_n

Graphic represantation of the probability function of the DRV is called polygon of disribution



2.2. Numerical characteristic of discrete random variables.

The DRV of X is completely described by means of the probabiliuty function. However , in many cases the probability function is unknown or it is enough to indicate the number expressing the most important properties of the distibution of X in compact form. These numbers are called characteristics of random variable. The most important characteristics are mean value, variance, standard deviation, cevariance.

2.2.1.Mean value (***mathematical expectation*** (мат ожидание), expected value, average)

The mean value of a DRV X is defined as the sum of productions of all possible values of the DRV to their probabilities:

$$M(X) \equiv \mu = x_1 p_1 + x_2 p_2 + \ldots + x_n p_n = \sum_{i=1}^{n} x_i p_i \qquad (10)$$

When number of experiences is tend to infinity the ariphmetic mean is tend to the expected value. The expectation is called also a center of gravity.

The properties of math expectation:

1^0 *M(C)=C*. The math expectation of constant value C is equal to the constant C.

2^0 The constant multiplier is taken out of the expectation. $M(C\cdot X)=C\cdot M(X)$

3^0 The expectation of algebraic sum of several DRVs is equal to algebraic sum of the expectations of the DRVs. $M(X\pm Y) = M(X)\pm M(Y)$

4^0 For independent random variables X and Y the expectation of product of the variables is equal to multiplication of math expectations $M(X\cdot Y)=M(X)\cdot M(Y)$.

2.2.2. Variance (разница) of discrete random variables.

A measure of describing variation about the mean is called a variance of the DRV.

$$D(X)=\sigma^2 = M(X^2) - \mu^2, \qquad (11)$$

Properties of variance:

1. The variance of a constant equals 0: $D(C) = 0$.
2. The constant multiplayer is taken out in square: $D(CX) = C^2 D(X)$
3. For independent random variables X and Y: $D(X\pm Y) = D(X)+D(Y)$

2.2.3 Standard deviation.

The unit of variance is the square of the unit X itself. The square root of the variance has the same unit and can be compared with the mean. This value is called a root-mean-square value or standard deviation:

$$\sigma(X)=\sqrt{D(X)}\ . \qquad (12)$$

§3. Continuous Random Variables

3.1. Cumulative disribution function.

It's impossible to describe the distribution function of a continuous random variable (CRV) by means of a table which would been contained all possible values of the variable and their probabilities. For CVR it is convenient to use the probability of the event "X<x"

Def: Disribution function of CVR is called the function F(x) being equal to the probability that the random variable would take the value less than x: F(x)=P(X<x). The function F(x) is called also the cumulative disribution functon (c.d.f.).

The properties of c.d.f.:

1. $0 \le F(x) \le 1$ as a probability;
2. F(x) is nondescreasing function;
3. The probability of hitting on interval (a,b) equals to P(a<x<b)=F(b)–F(a);
4. If the all possible values of CRV belongs the interval (a,b) than F(x)=0 for x≤a and F(x)=1 for x≥b;
5. If the all possible values of CRV belongs the number axis then $\lim\limits_{x\to-\infty} F(x)=0$ and $\lim\limits_{x\to+\infty} F(x)=1$.

3.2. Deusity function

Def: The density function of CRV is called the function f(x) be equal to the derivative of c.d.f.: **f(x)=F′(x)**.

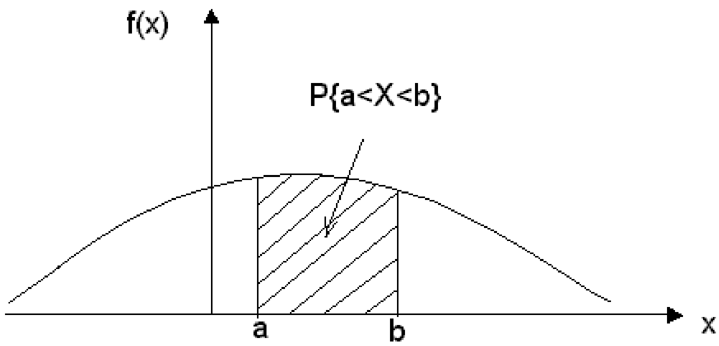
The function f(x) is also called differential distribution function, and the function F(x) is integral distribution function.

The properties of density function:

1. f(x) ≥ 0 as F(x) – nondecreasing function;
2. The probability of hitting an interval (a, b) equals to definite integral from **a** to **b**

$$P(a<x<b)=\int_a^b f(x)dx = F(b)-F(a) \qquad (13)$$

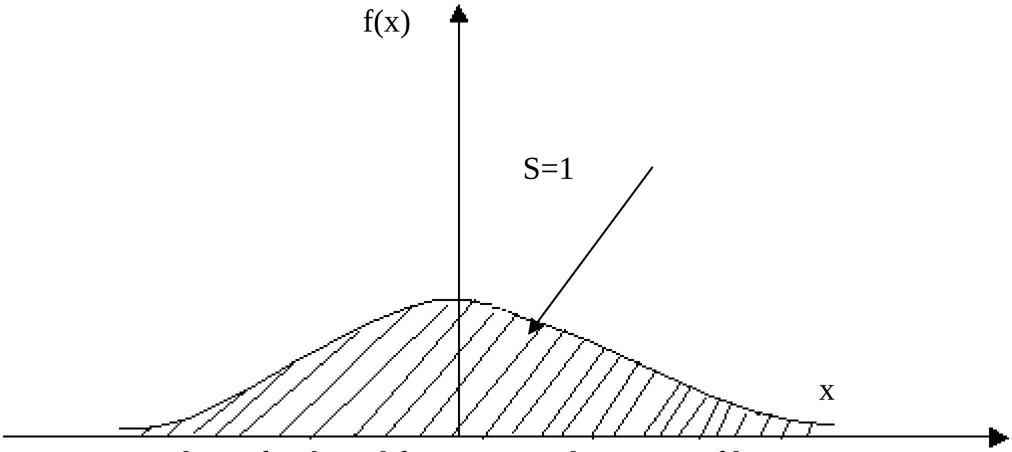
Graphically the probability of hitting the interval (a, b) equals the area of the curvilinear trapezoid in Fig:



3. The norming condition for the density function is expressed in terms of improper integral:

$$\int_{-\infty}^{+\infty} f(x)dx = 1$$

The area under graph of f(x) is equal to one (Fig)



The norming condition for the c.d.f. is expressed in terms of limit :

$$\lim_{x \to +\infty} F(x) = 1$$

3.3. <u>Numerical characteristics of conditions random variable</u>

For CRVs there are the same numerical characteristics as for DVRs where integration is used instead of summation

The <u>math expectation</u> of a CRV X is equal to

$$M(X) \equiv \mu = \int_{-\infty}^{+\infty} xf(x)dx \qquad (14)$$

In the case of the segment (a, b):

$$M(X) = \int_a^b xf(x)dx$$

The <u>variance</u> of a CRV X is equal to

$$D(x) = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)dx \qquad (15)$$

The <u>standard deviation</u> of a CRV X is equal to

$$\sigma(X) = \sqrt{D(X)} \qquad (16)$$

Example. Random variable is defined by cumulative distribution function

$$F(x) = \begin{cases} 0, & if \quad x < 0 \\ x^2, & if \quad 0 \le x < 1 \\ 1, & if \quad x \ge 1. \end{cases}$$

Find density function f(x), E(x), var(x) and probability P(0.5<X<1).

Solution.

f(x)=F'(x) from were $f(x)=\begin{cases}0, & if \quad x<0\\ 2x, & if \quad 0\le x<1\\ 0, & if \quad x\ge 1.\end{cases}$

$$M(X)\equiv\mu=\int_{-\infty}^{+\infty}xf(x)dx=\int_{-\infty}^{0}0dx+\int_{0}^{1}x\cdot 2xdx+\int_{1}^{+\infty}0dx=0+2\int_{0}^{1}x^2dx+0=$$

$$=2\left.\frac{x^3}{3}\right|_0^1=\frac{2}{3}$$

$$D(x)=\int_{-\infty}^{+\infty}(x-\mu)^2 f(x)dx=\int_{-\infty}^{+\infty}x^2 f(x)dx-\frac{4}{9}=\int_{0}^{1}x^2\cdot 2xdx-\frac{4}{9}=2\int_{0}^{1}x^3dx-\frac{4}{9}$$

$$=2\left.\frac{x^4}{4}\right|_0^1-\frac{4}{9}=\frac{1}{2}-\frac{4}{9}=\frac{1}{18}$$

$$\sigma(X)=\sqrt{D(X)}=\frac{1}{3\sqrt{2}}$$

$$P(a<x<b)=\int_a^b f(x)dx=F(b)-F(a)$$

$$P(0.5<X<1)=F(1)-F(0.5)=1-0.5^2=1-0.25=0.75$$

3. The purpose of the activities of students in class:
The student should know:
1. The concept of a random event, the types of random events.
2. Determination of the probability of a random event.
3. Basic probability theorems.
4. The concepts of discrete and continuous random variables.
5. Methods of assigning a random variable.
6. Basic numerical characteristics of a discrete and continuous random variable.

The student should be able to:
1. Apply the basic probability theorems, the Bernoulli and Poisson formulas.
2. To make the distribution law of a discrete random variable.
3. To calculate the basic numerical characteristics of a random variable.
4. Find the probability of hitting the value of a continuous random variable specified by the distribution function in a given interval.
5. Find the probability that the value of a normally distributed random variable falls within a specified interval.

4. The content of training:
Theoretical part:
1. Types of random events.
2. Classical and statistical determination of the probability of a random event.
3. Theorems of addition and multiplication of probabilities of random events.
4. Independent repeated tests (Bernoulli scheme). Bernoulli formula. Poisson formula.

5. The concept of a discrete random variable. Basic numerical characteristics of a discrete random variable and their properties.
6. The concept of a continuous random variable. Ways to set a continuous random variable.
7. The distribution function of a random continuous quantity, its properties, graph.
8. Density of the probability distribution of a continuous random variable, its basic properties.

The practical part:
1. Patients with four types of diseases are admitted to some hospitals. Long-term observations show that probabilities correspond to these types of diseases: 0.1; 0.4; 0.3; 0.2. For the treatment of diseases with a probability of 0.1 and 0.2, blood transfusion is necessary. How many patients need to be provided with blood if 1000 patients were admitted within a month?
2. The probability of getting hepatitis C for residents of a certain region in a certain period of the year is 0.0005. Assess the likelihood that of the surveyed 10,000 inhabitants 4 will be sick.
3. Six people have a disease for which the recovery rate is 98%. What is the probability that:
a) all six will recover;
b) only five will recover?
4. In a group of 15 students, 5 passed the colloquium in physics as excellent and 6 as good. What is the probability that a student selected from this group randomly passed the colloquium for "good" or "excellent"?
5. Taking the probability of a boy to be born at a birth of 0.5, find the probability that in a family with 6 children:
a) no boys;
b) 4 boys;
c) all children are boys.
6. Among the seeds of rye 0.4% weed seeds. What is the probability with a random selection of 5000 seeds to detect 5 weed seeds?
7. In the box 10 parts, among which 6 are painted. The collector randomly extracts 4 parts. Find the probability that all the details will be colored.
8. The student knows 20 of the 25 program questions. Find the probability that the student knows the three questions suggested to him by the examiner.
9. The number of pharmacists in each of the 20 pharmacies of the city is 3, 6,5,6,4,5,5,4,6,3,5,4,6,5,7,6,4,5,5, and 6 people. Create a law of the distribution of a random variable X, defined as the number of pharmacists in a randomly selected pharmacy (out of 20 pharmacies). Find the expectation, variance and standard deviation of this quantity.
10. Find the probability of hitting the interval (1.3) values of a continuous random variable given by the distribution function:

$$F(x) = \begin{cases} 0, if x \le 1, \\ \dfrac{x^2+1}{20}, if 1 < x \le 4, \\ 1, if x > 4 \end{cases}$$

5. The list of questions to check the initial level of knowledge:
1. What is called a random event?
2. Give definitions and examples of various types of random events (reliable, impossible, cooperative, incompatible, etc.).
3. What is the main characteristic of a random event?
4. Give the classical and statistical definitions of the probability of a random event.
5. What is the Bernoulli scheme called? Give the formulas of Bernoulli and Poisson.
6. Give the definition of a random variable.
7. Give the definition of a discrete random variable. Give examples.

8. Write the formulas of the main numerical characteristics of a discrete random variable.
9. Give the definition of a continuous random variable.
10. What is the probability density of a continuous random variable? How is it related to the distribution function?
11. Write down the formulas for the basic numerical characteristics of a continuous random variable.
12. Write down the formula for normal distribution.

6. The list of questions to check the final level of knowledge:

1. What approach to determining the probability of a random event (classical or statistical) requires real tests? Why?
2. Formulate theorems of addition and multiplication of probabilities of random events.
4. Give the properties of the basic numerical characteristics of a discrete random variable.
5. Why is it impossible to define a continuous random variable in the form of a table of its distribution law and using a formula?
6. Give the properties of the distribution function and the probability density of a continuous random variable**.**

7. Chronocard of the class:

1. Organizational moment - 5 min.
2. Analysis of the topic - 30 min.
3. Solving situational tasks - 60 min.
4. Current knowledge control - 35 min.
5. Summing up the lessons - 5 min.

8. List of textbooks for the lesson:

1. L.V. Kukharenko, O.V. Nedzved, M.V. Goltsev, V.G. Leshchenko, “Medical and biological physics for medical students”, Minsk BSMU 2016.

Topic: «MATHEMATICAL STATISTICS FUNDAMENTALS»

1. Scientific and methodological substantiation of the topic:

Mathematical statistics is a branch of mathematics devoted to mathematical methods of systematization, processing and use of statistical data. In this case, statistical data refers to information about objects of a sufficiently large aggregate with certain characteristics.

The healthcare professional should be able to interpret the results of laboratory tests, clinical measurements and observations, taking into account random fluctuations of physiological parameters, the possibility of observation error and the spread of instrument readings. Knowledge of statistical methods and the ability to apply them is necessary not only to understand the biomedical scientific disciplines, but also to work effectively in any area of health care. Such knowledge is necessary for understanding and interpreting biological, clinical and laboratory data.

2. Theory

1. General and selective statistical aggregates

The set of objects characterized by some qualitative or quantitative trait is called a statistical aggregate.

For example, the statistical aggregate is the set of solutions of chemical compounds, distinguished by both color (qualitative trait) and concentration (quantitative trait).

The statistical aggregate consisting of all objects that (at least theoretically) are subject to survey is called the general statistical aggregate. The number of objects in the population is called its volume and is designated *N*.

A statistical population consisting of a number of objects randomly selected from the corresponding general population is called a sample statistical population (sample). The number of objects in the sample is called its volume and denoted by n.

The randomness of selection is necessary so that the properties of the sampling set best reflect the corresponding properties of the general population, i.e. so that the sample was representative (representative).

2. Statistical discrete distribution series

Suppose you want to study the distribution of the values of the characteristic X for the objects of a certain general population.

For this purpose, a certain sample of volume n is extracted from the general population.

Let the smallest value of the attribute x_1 in the resulting sample set be m_1 times, the next largest value times, x_2 - m_2......., x_k - m_k times.

The observed values of the attribute are called variants, and the numbers m_1, m_2, m_3, ..., m_k - their frequencies.

Obviously, the sum of all frequencies is equal to the sample size:

$$m_1 + m_2 + \ldots m_k = \sum_{i=1}^{k} m_i = n, \tag{1}$$

The results of the observations are recorded in the form of a table, in the first line of which all the options x_i are listed in ascending order, in the second - the frequencies corresponding to them:

Table 1

X	x_1	x_2	…..	x_k
m	m_1	m_2	…..	m_k

Such a table is called a *statistical discrete series of distribution*.

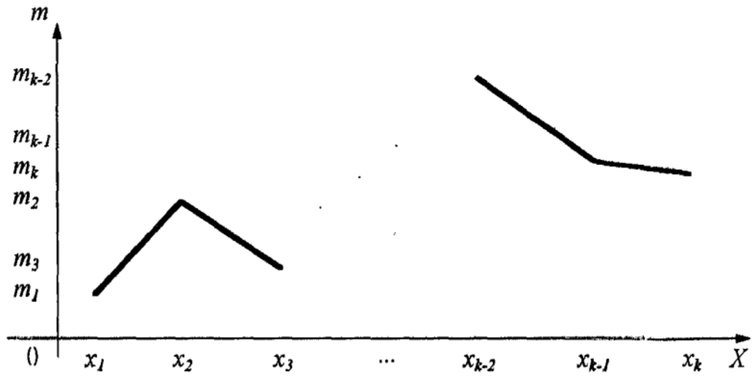


Fig.1

For a graphic image of such a series, points $(x_i;\ m_i)$ are laid on the coordinate plane and connected by straight line segments (Fig. 1). The resulting broken line, which is a graphic image of a discrete statistical series of the distribution, is called a frequency range.

Along with the frequencies m_i , the relative frequencies $p_i = \frac{m_i}{n}$, are often used, the sum of which is equal to one:

$$\sum_{i=1}^{k} p_i = 1, \tag{2}$$

Then, when constructing both the most discrete statistical series of the distribution, and its graphic image, called the polygon of relative frequencies, it is not the frequencies m_i that are used, but the relative frequencies p_i.

3. Statistical interval series of distribution

The use of a discrete distribution series in practice is convenient only in the case of a limited (no more than 10–20) number of differing variants in the sample. If the number of such options is significantly larger, then the results are presented as a *statistical interval series of distribution*.

To build such a series, the range of observed values of the trait under study is divided into a small number of equal-sized intervals, and the number of trait values in each interval (frequency of the interval) is recorded.

Let all observed values of the attribute X belong to the interval (a, b). We divide this interval into k partial intervals of length $\Delta x = \frac{b-a}{k}$ dividing points $a = x_0 < x_1 < x_2, \ldots, < x_{k-1} < x_k = b$ (Fig. 2):

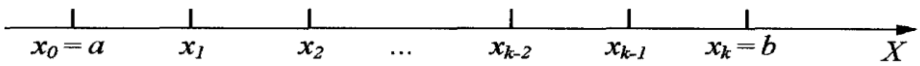


Fig. 2

Let's make a table, in the first line of which all partial intervals are listed, in the second - the frequencies corresponding to them (Table 2)

Table 2

X	(x_0,x_1)	(x_1,x_2)	(x_2,x_3)	…..	(x_{k-2},x_{k-1})	(x_{k-1},x_k)
m	m_1	m_2	m_3	…..	m_{k-1}	m_k

Such a table is called a statistical interval series of distribution, and its graphic image is a frequency histogram (Fig. 3).

The histogram of frequencies is a figure consisting of rectangles, the bases of which are partial intervals of length Δx, and the heights are the ratios $\frac{m_i}{\Delta x}$ (density of frequencies).

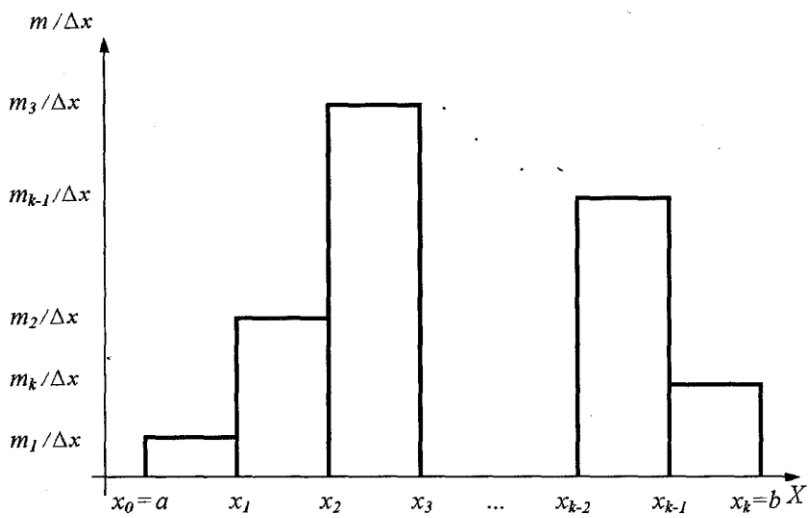


Fig. 3

In practice, often in the second line of the statistical interval series of the distribution, instead of the frequencies m_i, indicate the relative frequencies $p_i = \frac{m_i}{n}$.

Table 3

X	(x_0,x_1)	(x_1,x_2)	(x_2,x_3)	…..	(x_{k-2},x_{k-1})	(x_{k-1},x_k)
P	p_1	p_2	p_3	…..	p_{k-1}	p_k

Then the graphic image of such a series of distribution is a histogram of relative frequencies, while plotting which, on the ordinate axis, lay down the density of the relative frequency (Fig. 4).

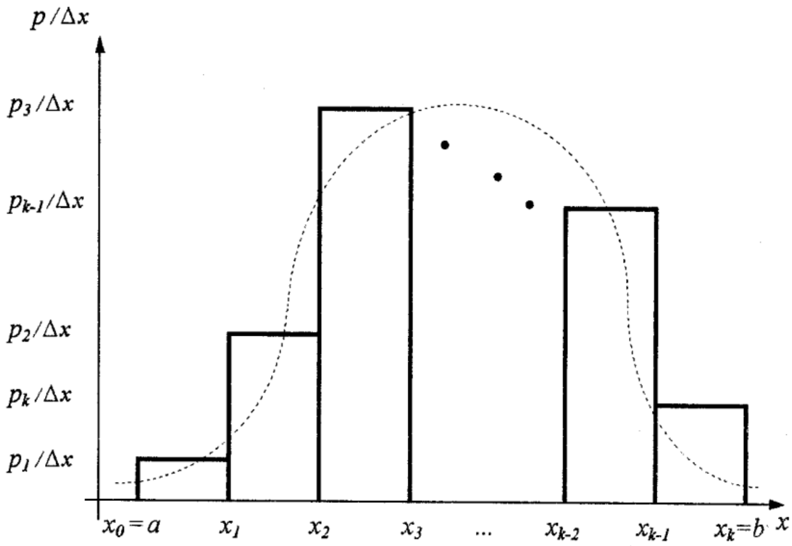


Fig. 4

When building a histogram, it is very important to choose the width of the interval correctly Δx. If the number of intervals k is small (the width of the interval Δx is large), it should hand, if k is too large (Δx -little), the processing of measurement results will be unnecessarily time-consuming, without giving a significant gain in information. Practice shows that rational choice of the number of intervals k depending on the sample size using the table 4:

Table 4

Sample size *(n)*	25-40	40-60	60-100	100-200	200
Number of intervals (k)	5-6	6-8	7-10	8-12	10-15

4. Point estimates of the main numerical characteristics of the general population.

The evaluation of the characteristic of the distribution is called the point one, if it is determined by one number, to which the estimated characteristic is approximately equal.

The best estimate of the general average $\overline{X}$ is the average sample, defined as the arithmetic average of all the values of the characteristic being studied in the sample:

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{k} m_i x_i \ , \qquad (3)$$

Where m_i - is the frequency of occurrence of the value x_i in the sample, k - is the number of options, n - is the sample size.

The mathematical expression of the fact that the average sample represents the best estimate of the general average is approximate

$$\overline{X} \approx \overline{x}, \qquad (4)$$

The best estimate of the general variance σ^2 is the so-called corrected sample variance, determined by the formula

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{k} m_i \left(x_i - \overline{x}\right)^2 \ , \qquad (5)$$

The mathematical expression of the fact that the corrected sample variance is the best estimate of the general variance is approximate equality

$$\sigma^2 \approx s^2, \tag{6}$$

The best estimate of the general standard deviation σ is the corrected sample standard deviation s, determined by the formula

$$s = \sqrt{s^2}, \tag{7}$$

The mathematical expression for the fact that the corrected sample standard quadratic deviation represents the best estimate of the general standard square deviation is the approximate equality

$$\sigma \approx s, \tag{8}$$

Example. When counting the number of leaves on each of the 20 indoor plants of a particular type, the following results were obtained: 11, 10, 9, 10, 7, 11, 11, 13, 10, 8, 12, 10, 9, 12, 9, 10, 8, 12, 11, 10. Give point estimates of the main numerical characteristics of the population.

Decision. The results show that the number of leaves on plants varies from 7 to 13. The value 7 occurs 1 time, the value 8 - 2 times, the value 9 - 3 times, etc. Thus, we can compile the following discrete distribution series:

X	7	8	9	10	11	12	13
M	1	2	3	6	4	3	1

By the formula (3) we find the average sample:

$$\bar{x} = \frac{1}{20}(1\cdot 7 + 2\cdot 8 + 3\cdot 9 + 6\cdot 10 + 4\cdot 11 + 3\cdot 12 + 1\cdot 13) \approx 10.$$

Thus, the point estimate of the general average $\overline{X}$ is:

$$\overline{X} \approx 10.$$

Using the value of the average sample $\bar{x}$, using formula (5), we find the corrected sample variance:

$$s^2 = \frac{1}{19}\left[1\cdot(7-10)^2 + 2\cdot(8-10)^2 + 3\cdot(9-10)^2 + 6\cdot(10-10)^2 + 4\cdot(11-10)^2 + 3\cdot(12-10)^2 + \right.$$
$$\left. +1\cdot(13-10)^2 \right] \approx 2{,}4$$

and in accordance with (6) we obtain a point estimate of the general variance σ^2:

$$\sigma^2 \approx 2{,}4.$$

Further, by the formula (7) we find the corrected sample standard quadratic deviation s:

$$s = \sqrt{s^2} \approx \sqrt{2{,}4} \approx 1{,}5$$

and in accordance with (8) we get :

$$\sigma \approx 1{,}5.$$

5. Interval estimates of the main numerical characteristics of the population

An estimate of a distribution characteristic is called interval if it is defined by two numbers — the boundaries of the interval containing the characteristic being evaluated.

Let the random variable X have a normal distribution with unknowns μ and σ.

In a number of tasks, it is required to find a suitable numerical value for a parameter μ and estimate its accuracy.

To give an idea of accuracy and reliability in mathematical statistics, the so-called confidence intervals corresponding to a given confidence probability are used.

The confidence probability (reliability) of an estimate of a numerical characteristic using a confidence interval is the probability that this characteristic is in a given interval.

The wider the confidence interval, the higher the corresponding confidence level, and vice versa: the greater the confidence level we want to ensure, the greater the corresponding confidence interval will be.

In pharmacy, medicine, and biology, the confidence level is assumed to be 0.95 or 0.99. Let us consider the method of finding the confidence interval for a given confidence probability when estimating the general average based on the results of sample observations. It is assumed that the trait under study in the general population is distributed according to the normal law. The method is based on using the Student's t-distribution for a random variable.

$$T = \frac{\bar{x} - \bar{X}}{s_{\bar{x}}}, \tag{9}$$

Where

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}, \tag{10}$$

- corrected standard deviation of the average sample.

The half-width of the confidence interval for the interval estimation of the general average for a given confidence probability is found by the formula.

$$\Delta x = t_{\gamma}(f) \cdot s_{\bar{x}} \;, \tag{11}$$

where $t_{\gamma}(f)$ - Student's coefficient for confidence probability γ and the number of degrees of freedom $f = n-1$. Then the interval estimate of the general average appears to be a confidence interval.

$$\left(\bar{x} - \Delta x; \bar{x} + \Delta x\right), \tag{12}$$

in which with a confidence probability γ is the general average.

3. The purpose of the activities of students in class:

The student should know:

1. Definitions of general and selective statistical aggregates.
2. The concept of a statistical discrete series of distribution.
3. The concept of a statistical interval number distribution.
4. The main numerical characteristics of a sample of statistical aggregate.
5. Point and interval estimates of the main numerical characteristics of the general population.
6. Student's distribution.

The student should be able to:

1. Build polygons of frequencies and relative frequencies.
2. Build a histogram of frequencies and relative frequencies.
3. To find point estimates of the main numerical characteristics of the general population.
4. Find the interval estimates of the numerical characteristics of the general population.

4. The content of training:

Theoretical part:

1. Definitions of general and elective statistical aggregates. Sample size.
2. Statistical discrete distribution series.
3. Polygons of frequencies and relative frequencies.
4. Statistical interval series of distribution.
5. Histograms of frequencies and relative frequencies.
6. Point estimates of the main numerical characteristics of the general population.
7. Interval estimates of numerical characteristics of the general population.

The practical part:

1. From the products manufactured by the pharmaceutical factory, 20 boxes of a certain drug were randomly selected, the number of tablets in which turned out to be 48, 52, 50, 49, 51, 50, 47, 50, 49, 50, 51, 52, 48, respectively, 51, 50, 47, 49, 46, 53, 50. Present this data in the form of a discrete statistical distribution series and construct a frequency range, as well as a range of relative frequencies.
2. The measurement of the weight of P 30 students gave the following results (in kg): 61, 67, 73.74, 80, 68, 69, 57, 88, 82, 70, 60, 75, 76, 90, 76, 75, 58 , 62, 79, 61, 69, 85, 82, 80, 66, 71, 82, 83, 80. Build a statistical interval series of the distribution of the value of P, as well as histograms of frequencies and relative frequencies.
3. When measuring blood pressure in randomly selected 30 patients of the clinic, the following results were obtained (in mm Hg): 151, 166, 133, 155, 179, 148, 143, 128, 138, 172, 163, 157, 158, 136, 169, 153, 142, 147, 134, 164, 167, 131, 152, 156, 161, 154, 149, 122, 176, 145. Present this data in the form of an interval statistical distribution series and construct a histogram of relative frequencies.
4. To give a point estimate of the general variance for a given distribution of the sample size n = 100

X	1250	1275	1280	1300
M	20	25	50	5

5. When counting the number of leaves on each of the 25 indoor plants of a particular type, the following results were obtained: 7, 12, 10, 11, 8, 9, 10, 7, 13, 12, 8, 9, 10, 12, 11, 11, 7, 8, 9, 12, 12, 13, 13, 8, 10. With a confidence level of $\gamma = 0.95$, give an interval estimate of the general average number of leaves on plants.

5. The list of questions to check the initial level of knowledge:

1. Give the definition of the statistical population.
2. What is the general statistical population and sample?
3. What are the options and their frequencies?
4. Give the definition of a statistical discrete series of distribution.
5. What is a frequency range?
6. What assessment of the distribution characteristic is called a point?
7. Write down the formulas for the general average, the sample average, the general variance, the corrected sample variance, the general standard deviation, the corrected selective standard deviation.
8. What is the interval estimate of a numerical characteristic?
9. Give definitions of confidence interval and confidence level.

6. The list of questions to check the final level of knowledge:

1. How is the relative frequency range constructed?
2. Describe the construction of the statistical interval series of the distribution.
3. What are histograms of frequencies and relative frequencies? (illustrate).
4. What are the advantages of the relative frequency histogram compared to the frequency histogram for a continuous feature?
5. What determines the value of the sample characteristics of the distribution?
6. Write down the Student's tread distribution formula for the random variable.
7. What is the corrected mean square deviation of the sample mean?
8. Write down the formula for the half-width of the confidence interval for the interval estimation of the general average for a given confidence probability.

7. Chronocard of the class:

1. Organizational moment - 5 min.
2. Analysis of the topic - 30 min.
3. Solving situational problems - 60 min.
4. Current knowledge control - 35 min
5. Summing up the lessons - 5 min.

8. List of textbooks for the lesson:

1. L.V. Kukharenko, O.V. Nedzved, M.V. Goltsev, V.G. Leshchenko, "Medical and biological physics for medical students", Minsk BSMU 2016.

Theme: "AUDIOMETRY"

1. Scientific and methodological substantiation of the topic:

Sound waves are one of the main sources of information about the world around us. Thanks to them we can talk to each other, hear each other and various sources of sound.

Sound is a source of information about the condition of the internal organs of man. For medicine, acoustic concepts are important for assessing auditory sensations; therefore, it is necessary to study the basic characteristics of sound: subjective (height, loudness and timbre) and objective (frequency, intensity, acoustic spectrum).

2. Theory:

Sound is the vibrational motion of particles of an elastic medium propagating in the form of longitudinal waves: gaseous, liquid or solid.

The term "sound" is also used to denote the sensation caused by the action of sound waves on the organs of hearing of man and animals. A person hears sounds from 16 Hz to 20,000 Hz.

Note that the physical concept of sound covers waves of both audible and inaudible range.

Sound with a frequency below the audible range is called infrasound, higher - ultrasound.

Sounds are divided into tones (musical sounds), noise and sonic booms. A tone is a tone that is a periodic process. If this process is harmonic, then the tone is called simple (for example, the sound of a tuning fork).

To complex tones include anharmonic fluctuations (for example, vowel sounds of human speech, sounds of musical instruments). Complex tone can be decomposed into simple. The lowest frequency of such a decomposition corresponds to the fundamental tone. The remaining harmonics (overtones) have frequencies equal to 2, 3, etc. A set of frequencies with an indication of their amplitude is called the acoustic spectrum. The spectrum of a complex tone is ruled (Fig. 1).

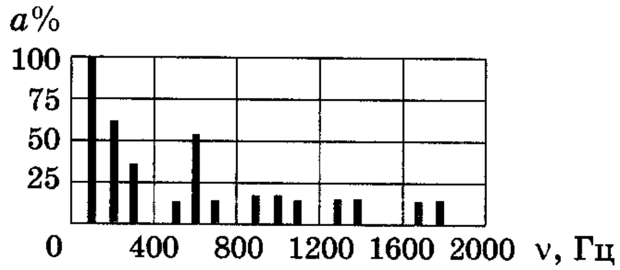


Fig. 1

A noise is a sound characterized by a complex, non-repeating time dependence (consonant sounds of speech, applause, sounds from machine vibration, etc.). Noise can be seen as a combination of randomly changing complex tones. The noise spectrum is continuous (Fig. 2).

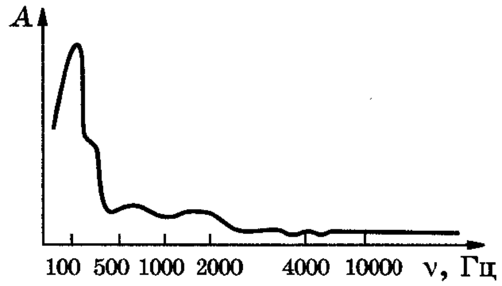


Fig. 2

Sonic impact is a short-term sound effect (for example, explosion, cotton).

The energy characteristic of sound as a mechanical wave is the intensity. But in practice it is more convenient to use sound pressure for sound evaluation, which additionally arises when sound waves pass through liquids or gases.

The intensity I and the sound pressure p are related by

$$I = \frac{p^2}{2\rho c}, \quad (1)$$

where ρ is the density of the medium, and c is the speed of sound.

In the auditory sensation, the height, loudness and timbre of sound are subjectively different. Each of these characteristics, in turn, depends on physical quantities that have an objective meaning: the frequency and intensity of the sound wave.

The pitch depends on the frequency of the oscillations. The higher the frequency, the higher the sound seems.

Musical sounds with the same basic tone are distinguished by a timbre, which is mainly determined by the frequencies and amplitudes of the overtones. We learn familiar voices and musical instruments precisely in timbre.

The sound volume depends on the intensity of the sound. The human ear is sensitive to sounds, the intensity of which varies in an incredibly wide range.

The lowest intensity of a sound wave that can be perceived by the hearing organs is called the threshold of audibility I_0.

The standard threshold of audibility is assumed to be equal to

$$I_0 = 10^{-12}\, W/m^2$$

at a fundamental frequency of 1 kHz.

The greatest intensity of a sound wave, in which the perception of sound does not cause pain, is called the threshold of pain sensation or the threshold of touch. The threshold of touch depends on the frequency of the sound and varies from 0, 1 W/m^2 at 6 kHz to 10 W/m^2 at low and sound frequencies.

Since the intensity range of the sounds we perceive is very large (the loudest sound that can perceive our ear has an intensity of 10^{13} times the intensity of the quietest sound we can still hear), it is convenient to compare the intensity of sounds in a logarithmic scale.

In this scale, the level of sound intensity is expressed in whites (B). If the level of a sound is 1B higher than that of another, then the intensity ratio of these sounds is 10. If the sound levels differ by 2B, then the ratio of their intensities 10^2, etc.В этой шкале уровень интенсивности звука выражается в *белах* (Б).

Usually, the intensity levels of the sounds are expressed in decibels (dB):

1 dB=0,1 B.

When constructing a scale of levels of sound intensity, the value is taken as the initial level of the scale; any other intensity is expressed in terms of the decimal logarithm of its ratio to I_0:

$$L_B = \lg \frac{I}{I_0}, \quad (2)$$

or when using decibels

$$L_{dB} = 10 \lg \frac{I}{I_0}, \quad (3)$$

The basis for creating a scale of loudness levels is the psychophysical law of Weber-Fechner: if the stimulation increases in a geometric progression (i.e., an equal number of times), then the sensation of this irritation increases in an arithmetic progression (i.e., the same value).

With respect to sound, this means that if the sound intensity takes a series of successive values, for example, aI_0, $a2I_0$, $a3I_0$, ... (a is some coefficient, a> 1), then the corresponding loudness sensations E_0, $2E_0$, $3E_0$, ...

Mathematically, this means that the sound volume is proportional to the logarithm of the sound intensity. If two sound stimuli with intensities I and I_0 are acting, and I_0 is the hearing threshold, then, based on the Weber-Fechner law, the loudness with respect to I_0 is related to the intensity as follows:

$$E = k\lg\frac{I}{I_0}, \qquad (4)$$

where k- is the proportionality coefficient. Strong dependence on frequency and intensity does not allow the measurement of loudness to reduce to the simple use of formula (4).

Conditionally believe that at a frequency of 1 kHz, the loudness and sound intensity scales completely coincide, and by analogy with (3)

$$E_{\phi} = 10\lg\frac{I}{I_0}, \qquad (5)$$

To distinguish it from the intensity scale in the volume scale, decibels are called backgrounds.

The volume at other frequencies can be measured by comparing the sound being studied with the sound at a frequency of 1 kHz.

In practice, the volume of sound can be estimated by the so-called curves of equal loudness, presented in Fig.3

Each of the presented curves combines sounds of the same loudness, measured in the backgrounds. It is assumed that the volume of any sound in the backgrounds coincides with the level of intensity of the sound (in decibels) at a frequency of 1 kHz: the audible threshold curve corresponds to the volume level 0 background; the nearest to it -10 background; next -20 background, etc. (see fig.3).

From the analysis of curves of equal loudness, it can be seen that two sound signals corresponding to the same intensity (for example, the sound of a frequency of 50 Hz and an intensity level of 70 dB and the sound of the same intensity level and frequency of 1000 Hz) have different levels of loudness - the first of the above sound the signal has a volume level of 30 background, the second-70 background.

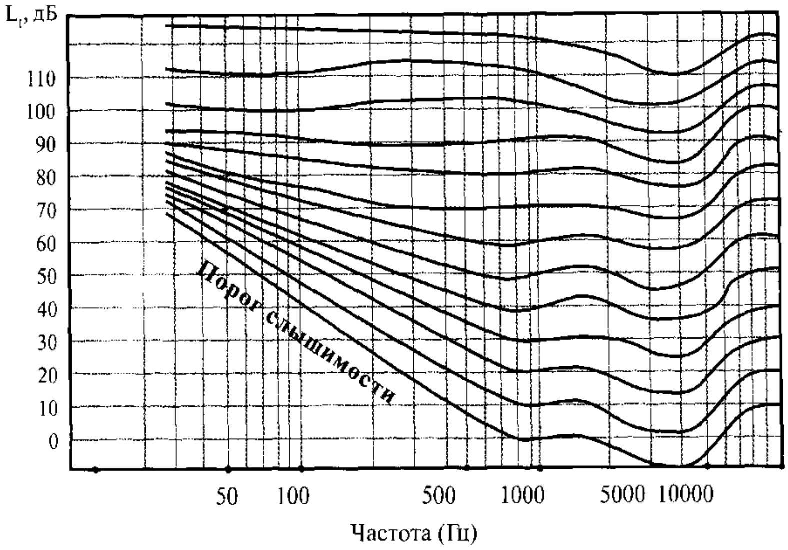


Fig.3

The method of measuring the severity of hearing is called audiometry: on a special device (audiometer), determine the threshold of auditory sensation at different frequencies; the resulting curve is called an audiogram. Comparing the patient's audiogram with a normal auditory threshold threshold, it is possible to diagnose a hearing disorder.

DESCRIPTION OF THE INSTALLATION

In the laboratory work, an audiometer AA-02 is used (Fig. 4).

Fig. 4

Audiometer AA-02 is designed to assess the functional state of a human hearing aid by determining the thresholds of audibility by air and bone sound by comparing the hearing of the subject with characteristics equivalent to the threshold of hearing the normal person, and by conducting above-threshold tests.

PERFORMANCE ORDER

Task 1. Removing the spectral characteristics of the ear at the threshold of audibility

1. Connect the phone, patient button and power cord to the corresponding connectors on the rear panel of the audiometer.
2. Connect the audiometer to the network.
3. Turn on the audiometer (the mains switch is located on the rear panel).
4. The following image will appear on the audiometer indicator:

ТОН: 1000 Hz	ПОДАЧА
ПРАВОЕ ВОЗД.	АВТОМАТ

5. Put the headband with the phones (on the right ear should be the "red" phone, on the left - "blue").
6. Press the RESET button.
7. Press the AUTO button.
8. Press the ANSWER button when there is sound in the phones. Release the button after each press.

ТОН: 1000Hz	50dB	***ПОДАЧА***
ПРАВОЕ ВОЗД.	АВТОМАТ	ТРЕНИРОВКА

9. After determining the thresholds of the right ear, the examination process is automatically repeated on the left ear with the same order of presentation of frequencies. The sound is indicated by asterisks on the indicator:
10. When the program for determining the air conduction thresholds is completed, an audible signal is heard in the audiometer and the audiometer automatically switches to the playback mode. The indicator displays the results of the survey, for example:

ЛЕВ.ВОЗД. Hz 125 250 500 750 1000 1500 ВОСПР. dB 45 35 30 --- 10 15

11. Conclude the results of your own surveys in the table:

Frequency Hz		25	250	500	750	1000	1500	2000	3000	4000	6000	8000
Threshold, dB	Left ear											
	Right ear											

12. Draw on the form of the audiogram (Fig. 5) the plot of the intensity of sound from the frequency for the right and left ear.

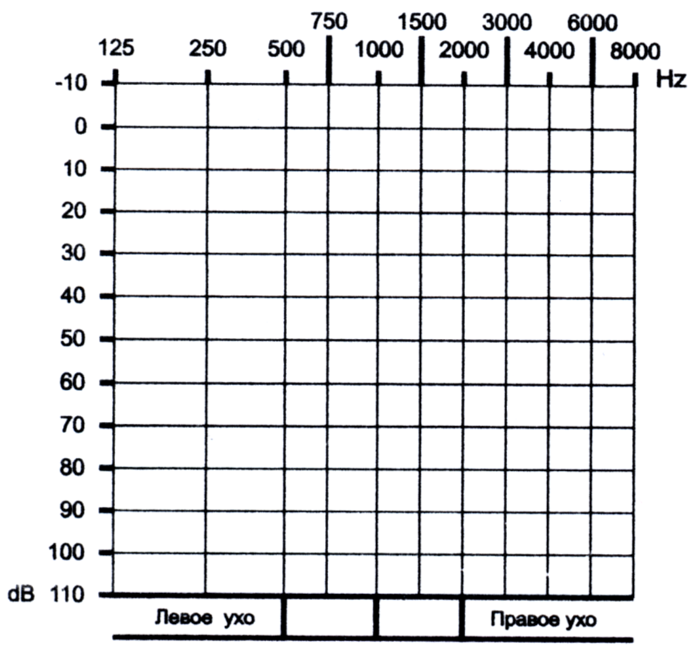


Fig. 5

Task 2. Solve situation problems:

1. Sound of what level of volume E, the background will hear a person, if it is hit by sound waves with a frequency of 1000 Hz and an intensity of 10-10 W / m^2?
2. Level of sound volume from one person at a frequency of 1000 Hz E = 40 background. What volume of sound E will create 30 simultaneously speaking people?
3. A single mosquito, located at a distance of 10 m from a person, creates a sound equal to the threshold of audibility I0 at a frequency of 1000 Hz. What level of volume E, will the background create 5000 mosquitoes that are at the same distance?
4. What is the sound intensity ratio I_1 / I_2, if the difference in intensity levels is ΔL = 40 dB?

5. Hearing loss in a patient at a frequency of 1 kHz is ΔL = 30 dB. Determine the minimum intensity of the I (W / m^2) wave, which at this frequency causes the patient to feel the sound.

3. The purpose of the activity of students in the class:
The student should know:
1. Types of sounds.
2. Objective and subjective characteristics of auditory sensation.
3. The Weber-Fechner law.
4. Units of the scale levels of intensity and volume of sound.

The student should be able to:
1. Explain the dependence of the physiological characteristics of the sensation of sound on the physical characteristics of the sound wave.
2. Build an audiogram.
3. To solve situational problems.

4. Training content:
1. Sound. Kinds of sounds: tones, noises, sonic booms. Simple and complex tone.
2. Acoustic spectrum. Sound pressure.
3. Characteristics of auditory sensation.
4. Construction of a scale of intensity levels.
5. The Weber-Fechner law.
6. Build a scale of volume levels.
7. Audiometry. Curves equal to the volume.
8. Removal of the spectral characteristics of the ear at the threshold of audibility.
9. The solution of situational problems.

5. List of questions to check the initial level of knowledge:
1. What is called sound? What are the types of sounds?
2. What is called simple and complex tone, noise, sonic boom?
3. What are the overtones?
4. List the physical characteristics of sound. In what units are they measured?
5. What is the acoustic spectrum? Give acoustic spectra of complex tone and noise.
6. What is called an audiogram?

6. List of questions for verifying the final level of knowledge:
1. Describe the methods for constructing scales of intensity levels and loudness of sound.
2. List the characteristics of the auditory sensation and indicate how they are related to the physical characteristics of the sound.
3. Formulate the Weber-Fechner law.
4. Describe the purpose and operating principle of the audiometer.
5. Describe the methods of auscultation, percussion and phonocardiography.

7. Independent work of students:
study the physical principles of sound methods of research in the clinic.

8. Chronocard training session:
1. Organizational moment - 5 min.
2. Knowledge control - 30 minutes.
3. Explanation of the work - 10 min.

4. Performance of work - 70 min.
5. Check work and homework - 20 min.

9. List of educational literature for the lesson:

1. L.V. Kukharenko, O.V. Nedzved, M.V. Goltsev, V.G. Leshchenko, “Medical and biological physics for medical students”, Minsk BSMU 2016.

Theme: "PHYSICAL BASIS OF ULTRASOUND RESEARCH"

1. Scientific and methodological substantiation of the topic:

Medical and biological applications of ultrasound are divided into two areas: diagnostic and research methods; methods of influence.

The first direction includes echoencephalography, ultrasound echolocation, ultrasound cardiography, measurement of heart sizes in dynamics. Using the ultrasonic Doppler effect, the rate of blood flow, the nature of the movement of the heart valves is studied.

The physical processes caused by ultrasonic exposure cause the following main effects in biological objects: microvibration at the cellular level; destruction of biomacromolecules; restructuring and damage to biological membranes; change in their permeability; thermal action; destruction of cells and microorganisms.

1.Theory:

1. Sources and receivers of ultrasound

Ultrasound (US) is called mechanical oscillations and waves, whose frequencies are more than 16-20 kHz.

The lower boundary of the region of ultrasonic frequencies that separates it from the region of audible sound is determined by the subjective properties of human hearing and is conditional, since the upper limit of the auditory perception of a person has a wide spread for different individuals. The upper limit of ultrasonic frequencies is due to the physical nature of the elastic waves, which can propagate only in a material medium, i.e. provided that the wavelength is much larger than the mean free path of molecules in gases or interatomic distances in liquids and solids. In gases, the limiting frequency is ~ 10^9 Hz, in liquids and solids ~ 10^{12}-10^{13} Hz.

Although the physical nature of ultrasound is the same as for sound waves of any frequency range, it has a number of specific features that determine its great importance in science and technology. These features are due to relatively high frequencies and correspondingly small wavelengths.

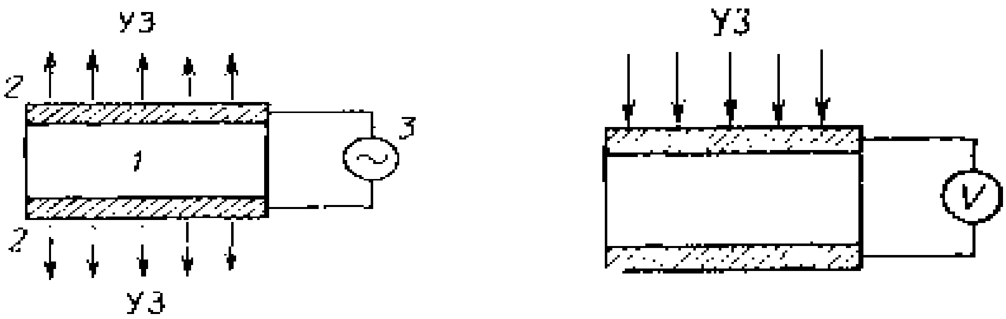
The smallness of the wavelength determines the radial nature of the propagation of ultrasonic waves. Near the radiator, ultrasonic waves propagate in the form of beams, the transverse dimension of which remains close to the size of the radiator. Getting to large obstacles or inhomogeneities in the environment, such an ultrasonic beam is reflected and refracted. When a beam hits a small obstacle or a defect, a scattered wave arises. This makes it possible to detect very small inhomogeneities in the medium, on the order of tenths and hundredths of a millimeter.

In nature, ultrasound occurs both as a component of many natural noises, and among the sounds of the animal world. Some animals use ultrasonic waves to detect obstacles (bats, dolphins, some bird species inhabiting dark caves). The ability to emit and perceive ultrasonic waves have some insects (crickets, cicadas, certain types of butterflies). As a rule, animals use for locating frequencies from tens to hundreds of kHz. Some mammals, such as dogs, cats, also have the ability to perceive ultrasound at a frequency of up to hundreds of kHz.

Artificial ultrasound emitters are based on the phenomenon of magnetostriction (at lower frequencies) and the inverse piezoelectric effect (at higher frequencies). Magnetostriction consists of imperceptible to the eye oscillations (elongation and shortening) of the length of the ferromagnetic core under the action of an alternating magnetic field in accordance with the frequency of change of the sign of the field.

Of the artificial ultrasound emitters, electromechanical radiators, based on the phenomenon of the reverse piezoelectric effect, are most widely used, which consists in the mechanical deformation of bodies under the action of an electric field. The main part of such a radiator (Fig. 1, a) is a plate 1 of matter with well-defined piezoelectric properties (quartz, barium titanate, etc.). Electrodes 2 are deposited on the surface of the plate as conductive layers. If an alternating voltage is applied to the electrodes from the generator 3, the plate will begin to vibrate due to the inverse piezoelectric effect, emitting a mechanical wave of the corresponding frequency.

The greatest effect of radiation of a mechanical wave occurs when the resonance condition is satisfied.



b)

a)

Fig. 1

The ultrasonic receiver can be created on the basis of a piezoelectric effect (direct piezoelectric effect). In this case, under the action of the ultrasonic wave, there is a deformation of the crystal (Fig. 1, b), which leads, under the piezoelectric effect, to the generation of an alternating electric field; the corresponding electrical voltage can be measured.

2. The effect of US on the substance

Ultrasound has a complex effect on the substance: mechanical, physicochemical and thermal.

The mechanical action of ultrasound on the substance is associated with the deformation of the microstructure of the substance, which is due to the alternate approach and rarefaction of its particles caused by ultrasonic waves. With sufficient intensity of ultrasound, this can cause destruction of the substance.

An important property of ultrasound is that it causes intense vibrational motion of the fluid particles (at each point of the phase the compaction alternates with the phases of rarefaction of the medium). Sometimes in such a liquid, discontinuities of the continuity of the medium (cavitation) occur and in the rarefaction phase microcavities are formed, which quickly fill with a saturated vapor of the surrounding liquid. The lifetime of the cavity (bubble) is very small, since in the wave following the rarefaction compression quickly comes on and the pressure on the bubble on the side of the surrounding liquid sharply increases (it may exceed the atmospheric pressure by several thousand times), which leads to collapse of the cavity and formation of strong percussion waves. This, in particular, is used to destroy the envelopes of plant and animal cells and extract biologically active substances from them.

The action of ultrasound can grind and disperse different media, which is used in the manufacture of vaccines, emulsions, aerosols, etc.

Depending on the conditions of exposure and the properties of the environment, ultrasound can contribute to the reverse processes: sedimentation of suspensions, coagulation of aerosols, and purification of gases from impurities suspended therein.

US accelerates some chemical reactions, for example, oxidation and polymerization.

On the complex action of mechanical, thermal and chemical factors, the biological effect of ultrasound is based, which can cause the death of viruses, bacteria and fungi, and with considerable power and small animals. At low power, US increases the permeability of cell membranes, activates the processes of tissue metabolism.

3. Application of ultrasound

The diverse applications of ultrasound, in which various features of the ultrasound are used, can be conditionally divided into three directions.

The first direction is connected with obtaining information by means of ultrasonic waves, the second - with active influence on the substance, the third - with the processing and transmission of signals.

Ultrasound methods are widely used in scientific research to study the properties and structure of substances, to elucidate the processes occurring in them at macro and micro levels. These methods are based on the dependence of the propagation velocity and attenuation of acoustic waves on the properties of substances and on the processes occurring in them.

The study of the propagation of ultrasound in crystals gives information on the features of the structure of the crystal lattice.

The elastic and strength characteristics of metals, ceramics, concrete, the degree of purity of materials, and the presence of impurities are determined from the ultrasonic velocity. The accuracy of determining the composition of substances, the concentration of impurities by ultrasonic methods is high and is a fraction of a percent.

By the change in the speed of sound or by the Doppler effect in moving liquids and gases, the rate of their flow is determined.

The use of acoustic location in hydroacoustics is of exceptional importance, since sound waves are the only type of waves propagating over long distances in the natural aquatic environment. Sonar instruments - echo sounders, sonars are used for navigation, in fishing, in naval affairs, in oceanological research.

Of great importance is the use of ultrasonic waves to detect hidden defects in materials and products - ultrasonic flaw detection, which is widely used in industry.

In the pharmaceutical industry, the ability of ultrasound to crush bodies placed in a liquid and create emulsions is used in the manufacture of drugs.

In the treatment of diseases such as tuberculosis, bronchial asthma, upper respiratory catarrh, use aerosols of various drugs obtained by ultrasound.

In medicine, ultrasound in different frequency ranges is used for therapeutic and surgical treatment and diagnostics.

The first application of ultrasound in medicine dates back to the early 1930s.

The treatment method, which uses oscillations in the range of 800-3000 kHz, is called ultrasound therapy.

For therapeutic purposes, apparatuses are used in which an electric current is supplied to a plate of a radiator - quartz or barium titanite. The plate under the action of an alternating electric field changes its volume - it is compressed and decompressed. The movements of the plate through the contact medium (vaseline oil, water) are transferred to the underlying tissues. Ultrasound in these frequencies is propagated in the media with an almost rectilinear beam, which allows them to act on a limited area and penetrates to a depth of 1 to 5-6 cm. This makes it possible to use it for the treatment of diseases of various organs: it is more absorbed by the muscles, from the bones 40-60% of the incident energy is reflected, it does not spread in the air.

In the therapeutic effect of ultrasound, mechanical (micromassage of cells and tissues), thermal (heat generation) and physicochemical (formation of biologically active substances) factors are isolated.

In medical practice, use mainly small doses of ultrasound, activating intracellular processes in tissues (protein biosynthesis, enhancement of enzyme activity, etc.). Therapeutic doses of ultrasound have a pronounced analgesic, vasodilating, anti-inflammatory effect.

Under the influence of ultrasound in the area of exposure, the permeability of the skin and mucous membranes increases, which facilitates the introduction of medicated medicines into the tissues through the skin. This method of treatment is called phonophoresis.

An apparatus for ultrasonic welding and cutting of bones has been created. In fractures, welding firmly connects the bone fragments and does not violate the natural processes of bone repair. Welding is also used to fill bone defects. In surgical interventions, ultrasound instruments are used to cut bones and soft tissues.

The ability of ultrasonic waves without significant absorption to penetrate into the soft tissues of the body and be reflected from seals and inhomogeneities is used for diagnostic purposes. Ultrasound diagnosis supplements the basic method of examining internal organs - X-ray diagnostics, and sometimes it has significant advantages over it. The almost complete absence of any side effects makes it possible to conduct multiple ultrasound examinations of any parts of the body, including fetal examination during pregnancy.

High sensitivity of ultrasound equipment allows you to receive an echogram of soft tissues, track moving objects, for example, the heart rate, the speed of blood flow in large vessels. With the help of ultrasound, the sizes of internal organs and their parts, tumors, hemorrhages, foreign bodies, stones, etc. are determined quite accurately. This causes the method to be widely recognized and implemented in many fields of medicine: surgery, ophthalmology, obstetrics and gynecology, oncology, sports and space medicine.

In diagnostics based on the echo method, frequencies of ~ 107 Hz are used. The intensity of ultrasound does not exceed 0.5 mW / cm^2, which is considered safe for the body.

4. The Doppler Effect

The Doppler effect is the change in the frequency of waves perceived by the observer due to the relative motion of the wave source and the observer.

When the source of waves and the observer are approached simultaneously, the frequency

$$\nu_p = \frac{c + \upsilon_{ob}}{c - \upsilon_s} \nu_s , \tag{3}$$

With simultaneous removal of the source of waves and the observer, the frequency

$$\nu_p = \frac{c - \upsilon_{ob}}{c + \upsilon_s} \nu_s , \tag{4}$$

where c- is the velocity of propagation of ultrasonic waves, $\upsilon_{ob} -$ is the speed of the observer, $\upsilon_s -$ is the speed of the source, $\nu_s -$the frequency of the source radiation, and $\nu_p -$ the frequency of the perceived waves.

The Doppler effect is used to determine the rate of blood flow.

A technical system containing a generator of 1 electric oscillations of the ultrasonic frequency is used (Fig. 2), a radiator US-2, a frequency comparison device 3. The ultrasonic wave 4 penetrates into the blood vessel 5 and is reflected from the moving erythrocytes 6. The reflected ultrasound wave 7 to the receiver 8, where it is converted into an electrical oscillation and amplified.

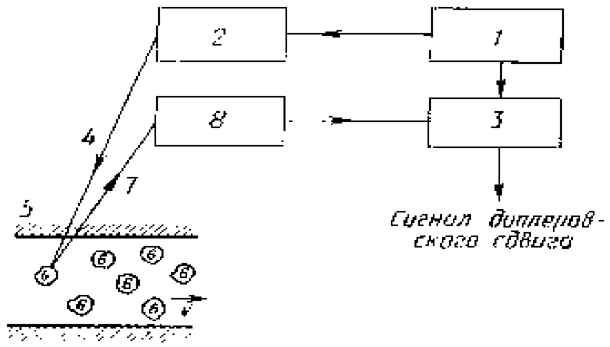


Fig. 2

If the generator emits ultrasound that falls on the erythrocyte, then, and using the formula (4), we have

$$\nu' = \frac{c+\upsilon_r}{c-\upsilon_s}\,\nu_s = \frac{c+\upsilon_0}{c}\nu_g\,, \tag{5}$$

where ν'- is the frequency of the waves perceived by the receiver (in this case, the erythrocyte).

Next, consider the ultrasound wave reflected from the erythrocyte, which will now become the source of ultrasound: $\upsilon_r = 0$, $\upsilon_s = \upsilon_{er} = \upsilon_0$, then

$$\nu'' = \frac{c}{c-\upsilon_0}\nu' = \frac{c}{c-\upsilon_0}\frac{c+\upsilon_0}{c}\nu_g = \frac{c+\upsilon_0}{c-\upsilon_0}\nu_g\,, \tag{6}$$

where ν''- is the frequency perceived by the receiver.

The frequency shift is:

$$\Delta\nu_D = \nu'' - \nu_g = \frac{c+\upsilon_0}{c-\upsilon_0}\nu_g - \nu_g = \nu_g\left(\frac{c+\upsilon_0}{c-\upsilon_0} - 1\right) = \nu_g\left(\frac{c+\upsilon_0-c+\upsilon_0}{c-\upsilon_0}\right) == \nu_g\,\frac{2\upsilon_0}{c-\upsilon_0}\,,$$

As c >>υ_0, then

$$\Delta\nu_D = \frac{2\upsilon_0}{c}\nu_g\,, \tag{7}$$

and the blood flow velocity is

$$\upsilon_0 = \frac{c\Delta\nu_D}{2\nu_g}\,, \tag{8}$$

$\Delta\nu_D$ is called the Doppler frequency shift.

3. The purpose of the activity of students in the class:
The student should know:
1. The physical nature of ultrasonic waves.

2. Methods of obtaining ultrasound.
3. Application of ultrasound in medicine and pharmacy.
4. The method of ultrasonic echolocation.

The student should be able to:

1. Derive a formula for determining the rate of blood flow.
2. Explain the method of obtaining ultrasound using the phenomenon of the inverse piezoelectric effect.
3. Explain the method of ultrasound echolocation.
4. To solve situational problems.

4. Training content:

1. The boundaries of ultrasonic frequencies.
2. Ultrasound in nature.
3. Artificial ultrasound sources.
4. Effect of ultrasound on the substance.
5. Use of ultrasound in medicine.
6. The effect of ultrasound on biological objects.
7. Ultrasound echolocation.
8. The Doppler effect. The use of the Doppler effect in medicine.
9. The solution of situational problems.

5. List of questions to check the initial level of knowledge:

1. What is ultrasound? What determines the boundaries of ultrasound frequencies?
2. What specific features does ultrasound have?
3. Give examples of ultrasound in nature.
4. How to get ultrasound?
5. How does ultrasound affect the substance?
6. What is called the Doppler effect?

6. List of questions for verifying the final level of knowledge:

1. Explain the phenomenon of the inverse piezoelectric effect.
2. What is the phenomenon of magnetostriction?
3. Explain the essence of the Doppler effect for mechanical waves.
4. How is ultrasound used for diagnostic and therapeutic purposes?
5. Describe the method of ultrasound echolocation.
6. Display a formula for determining the blood flow velocity.

7. Situational tasks:

1. In determining the blood flow velocity, the Doppler frequency shift was $\Delta\nu$ / ν = 0.06%. Determine the blood velocity, if the ultrasound velocity in it is c = 500 m / s.
2. The erythrocyte moves in the blood stream at a velocity υ = 300 mm / s. It falls on and then reflects an ultrasonic wave from a stationary source (probe) operating at a frequency of 5 MHz. Determine the frequency difference $\Delta\nu$ between the reflected erythrocyte and the radiated source by ultrasonic waves if the erythrocyte is removed from the source. The velocity of ultrasound in the blood take c = 1500 m / s.
3. The erythrocyte moves in the blood stream at a velocity υ = 200 mm / s. It falls on and then reflects an ultrasonic wave from a stationary source (probe) operating at a frequency of 6 MHz. Determine the frequency difference $\Delta\nu$ between the reflected erythrocyte and the radiated source by ultrasonic waves, if the erythrocyte approaches the source. The velocity of ultrasound in the blood take c = 1500 m / s.

4. Calculate the maximum speed and acceleration of air particles in an ultrasonic wave at a frequency of 100 kHz, if the amplitude of the particle displacement is 0.5 mm.
5. The speed of the ultrasound in the skull bone is 3500 m / s, the frequency is 3 MHz. Determine the resolution limit of the ultrasound analyzer if the number of periods in the parcel is 9.
6. Determine the minimum frequency of ultrasonic waves in the premise of 8 periods, if in the study of bone it is required to distinguish a 2 mm crack, c = 3500 m / s.

8. Chronocard training session:
1. Organizational moment - 5 min.
2. Analysis of the topic - 30 min.
3. The solution of situational problems - 60 min
4.Tekushchy knowledge control - 35 minutes.
5. Summing up the lessons - 5 min.

9. List of educational literature for the lesson:
1. L.V. Kukharenko, O.V. Nedzved, M.V. Goltsev, V.G. Leshchenko, “Medical and biological physics for medical students”, Minsk BSMU 2016.

Theme: "DETERMINATION OF THE LIQUID VISCOSITY"

1. Scientific and methodological substantiation of the topic:

Liquid media make up the largest part of the body, their movement ensures the exchange of substances and the supply of cells with oxygen, so the mechanical properties and flow of liquids are of interest to physicians.

When the fluid moves, including blood through the blood vessels, an important role is played by internal friction or viscosity.

2. Theory:

In a real liquid, between the molecules, the forces of mutual attraction act, which determine the internal friction (viscosity). Internal friction, for example, causes resistance force when stirring the liquid, slowing down the rate of falling of bodies thrown into the liquid, etc.

Newton established that the force F of internal friction between two layers of fluid moving with different velocities (Fig. 1) depends on the nature of the liquid and is directly proportional to the area S of the contacting layers and to the velocity gradient $\frac{d\upsilon}{dz}$ between them:

$$F = \eta S \frac{d\upsilon}{dz} , \qquad (1)$$

where η -is the coefficient of proportionality, called the viscosity coefficient or simply the viscosity of the fluid and depending on its nature.

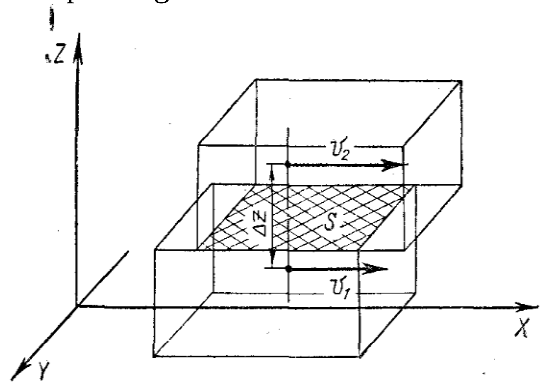


Fig. 1

The force F acts relative to the surface of the contacting layers of the fluid, accelerates the layer moving more slowly and slows the layer moving more rapidly.

The velocity gradient characterizes the rate of change of velocity between the layers of the liquid, i.e. in a direction perpendicular to the direction of flow of the liquid.

The unit of the viscosity coefficient in the SI system is N·s / m^2, in CGS - dyne · s / cm^2, this unit is called Poise (P):

$$1P=0,1\ N\cdot s / m^2$$

In most liquids (water, low-molecular compounds, true solutions, molten metals and their salts), the viscosity coefficient depends only on the nature of the liquid and temperature: with increasing temperature, the viscosity coefficient decreases. Such liquids obey equation (1) and are called Newtonian, and their viscosity is called normal. In most liquids (water, low-molecular compounds, true solutions, molten metals and their salts), the viscosity coefficient depends only on the nature of the liquid and temperature: with increasing temperature, the viscosity coefficient decreases. Such liquids obey equation (1) and are called Newtonian, and their viscosity is called normal.

In practice, the relative viscosity is used η_r, which refers to the ratio of the viscosity coefficient of a given liquid to the water viscosity coefficient at the same temperature**:**

$$\eta_r = \frac{\eta}{\eta_w} 100\,\% , \qquad (2)$$

In high-molecular liquids (for example, polymer solutions) or representing disperse systems (suspensions and emulsions), the viscosity coefficient also depends on the flow regime-pressure and velocity gradient. Such liquids do not obey equation (1), are called non-Newtonian, and their viscosity is called anomalous. For example, non-Newtonian fluids include blood.

With a relatively low velocity of flow through small diameter pipes, the movement of the liquid is laminar: a layer of molecules adheres to the wall of the pipe adjacent to the wall and remains immobile, the next layer under the action of the pressure force and against the force of internal friction between the layers is displaced relative to the wall layer and moves with a small speed. Each successive layer of molecules, moving relative to the previous layer, moves relative to the wall of the tube with a constantly increasing velocity, which reaches its highest value at the center of the tube (Fig. 2):

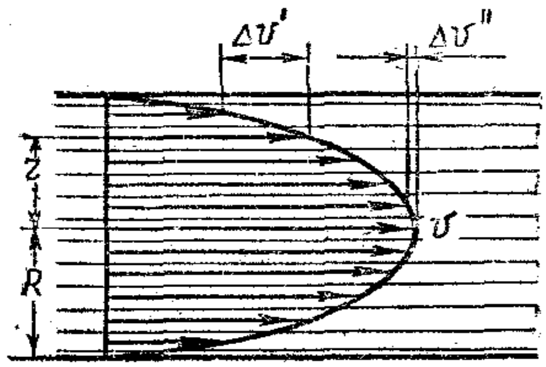


Fig. 2

The distribution of velocities along the section of a circular pipe is parabolic (Fig. 2):

$$\upsilon = \frac{p_1 - p_2}{4l\eta}(R^2 - r^2), \qquad (3)$$

where p_1 and p_2 -are the pressures at the beginning and end of a pipe section of length l, η-is the fluid viscosity coefficient, R- is the pipe radius, and r- is the radius of the liquid layer in question. The maximum speed is observed in the center of the pipe.

$$\upsilon_{\max} = \frac{p_1 - p_2}{4l\eta} R^2, \qquad (4)$$

Laminar flow is established in pipes with smooth walls, without sudden changes in the cross-sectional area or bends of the pipe, in the absence of multiple branches.

If these conditions are violated, and especially at high speeds, the flow becomes turbulent.

Characteristic for turbulent flow are local changes in pressure in the liquid, accompanied by sound phenomena (noise, murmur).

The velocity υ_{cr} of the laminar flow transition into a turbulent flow is determined by the Reynolds number Re:

$$Re = \frac{\upsilon_{cr}\rho D}{\eta} \quad , \tag{5}$$

where ρ -is the density of the liquid, D- is the diameter of the tube.

For a straight smooth pipe Re_{cr} = 2300. If $Re \geq Re_{cr,}$ then the flow goes into turbulent flow.

The average velocity of a laminar flow along a narrow horizontal pipe of constant cross section is equal to (Poiseuille's law):

$$\upsilon_{av} = \frac{p_1 - p_2}{8l\eta}R^2 \quad , \tag{6}$$

where p_1 and p_2 -are the pressures at the beginning and end of a pipe section of length l, η -is the fluid viscosity coefficient, R- is the pipe radius.

The amount of liquid Q flowing through the cross section per unit time is equal to $Q = \upsilon_{av}S$, where $S = \pi R^2$. Consequently

$$Q = \frac{(p_1 - p_2)}{l}\frac{\pi R^4}{8\eta} \quad , \tag{7}$$

The formula (7) is called the Hagen-Poiseuille formula. It can be written in the form

$$Q = \frac{p_1 - p_2}{X}, \tag{8}$$

where $X = \dfrac{8l\eta}{\pi R^4}$ called the hydraulic resistance.

To measure the relative coefficient of viscosity of not very viscous liquids, a capillary viscometer is used.

For more viscous liquids, a method based on the measurement of the velocity of incidence in a fluid of small bodies of spherical shape (the Stokes method) is used.

Stokes empirically established that, when the body of a spherical shape (ball) is not too fast, the resistance to movement is

$$F = 6\pi r\eta\upsilon , \tag{9}$$

where r- is the radius of the ball, υ- is the velocity of motion, η- is the viscosity coefficient of the fluid.

When the ball falls in a narrow cylinder filled with the test liquid, three forces act on the ball (Fig. 3): the resistance force $F=6\pi r\eta\upsilon$, the gravitational force $P = \frac{4}{3}\pi r^3\rho_b g$ and the buoyancy force $F_A = \frac{4}{3}\pi r^3\rho_l g$, where ρ_b - is the density of the ball substance, ρ_l -is the density of the liquid .

When the ball moves uniformly

$$P = F_A + F .$$

From here

$$F = P - F_A,$$

or

$$6\pi r \eta \upsilon = \frac{4}{3}\pi r^3 (\rho_b - \rho_l) g$$

and for the viscosity coefficient we obtain:

$$\eta = \frac{2}{9} g r^2 \frac{\rho_b - \rho_l}{\upsilon}, \qquad (10)$$

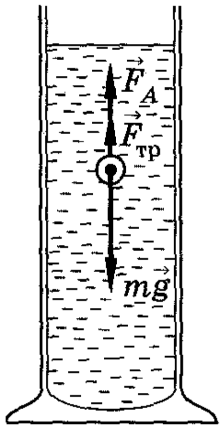


Fig. 3

Measuring the time for which the ball passes a certain distance, you can calculate the speed of its movement

$$\upsilon = \frac{l}{t}.$$

Then the final formula for determining the coefficient of viscosity takes the form:

$$\eta = \frac{2}{9} g r^2 \frac{\rho_b - \rho_l}{\ell} t, \qquad (11)$$

PERFORMANCE ORDER

To determine the coefficient of viscosity of the liquid, a graduated cylinder (Fig. 4), a stopwatch and 5-10 balls are used.

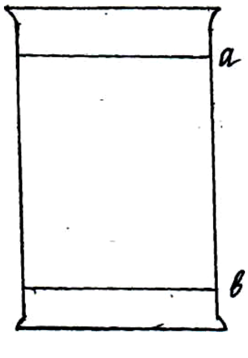


Fig. 4

a - the upper mark (rubber ring),
в - lower mark (rubber ring),

ρ_b = 11,4 · 10^3 kg / m^3 - density of the ball (lead),
ρ_l = 1,26 · 10^3 kg / m^3-density of the liquid being studied,
r = 1 · 10^{-3} m is the radius of the ball.

The task. Determination of the viscosity of a liquid
1. Moving the mark "a" and "in", set them at a certain distance.
2. Measure the distance between the marks and place the value in the table.
3. Throw the ball into the graduated cylinder.
4. Observe the process of falling the ball in the cylinder, mark by the stopwatch the time t of the passage of the ball by the distance.
5. Do the experiment 5-10 times.
6. Record the experience data in a table and perform the calculations.
7. Calculate the absolute and relative errors in the measurements.

Table

№	ℓ ,m	t,s	η,Pa ·s	$\Delta\eta$, Pa ·s	$\frac{\overline{\Delta\eta}}{\overline{\eta}}\cdot 100\%$
1. 2...					

$\overline{\eta}$ = $\overline{\Delta\eta}$ =

3. The purpose of the activity of students in the class:
The student should know:
1. The physical nature of the viscosity of a liquid.
2. Units of measurement of viscosity.
3. Newton's formula.
4. Concepts of laminar and turbulent flows.
5. The Hagen-Poiseuille formula.
6. Stokes method.

The student should be able to:
Determine the viscosity of the liquid by the Stokes method.

4. Training content:
1. Viscosity of the liquid. Newton's formula. Gradient of speed.
2. Coefficient of internal friction. Units of viscosity. Relative viscosity.
3. The flow of liquid through the pipes. Laminar flow. Formula for the speed of the layer. Distribution of the velocity of the layers along the pipe cross-section.
4. Turbulent flow. Reynolds number.
5. The Hagen-Poiseuille formula. Hydraulic resistance.
6. Determination of the coefficient of viscosity of liquids by the Stokes method.

5. List of questions to check the initial level of knowledge:
1. What is called internal friction?
2. Write down Newton's equation.
3. What is the velocity gradient?
4. In what units is the viscosity coefficient measured?

5. What is the relative coefficient of viscosity?
6. Which liquids are called Newtonian? non-Newtonian?

6. List of questions for verifying the final level of knowledge:

1. Describe the laminar and turbulent flow of a fluid.
2. What is the Reynolds number, from what values does it depend?
3. Obtain the Hagen-Poiseuille formula.
4. Dive out the formula for determining the viscosity of the liquid by the Stokes method.
5. What conditions must be satisfied when measuring the viscosity by the Stokes method?

7. Chronocard training session:

1. Organizational moment - 5 min.
2. Current knowledge control - 40 min.
3. Explanation of the work - 10 min.
4. Performance and execution of work - 60 min.
5. Check work and homework - 20 min.

8. List of educational literature for the lesson:

1. L.V. Kukharenko, O.V. Nedzved, M.V. Goltsev, V.G. Leshchenko, “Medical and biological physics for medical students”, Minsk BSMU 2016.

Topic: "REFRACTOMETRY"

1. Scientific and methodological substantiation of the topic:

The refractive index is an important optical characteristic of transparent media. It, for example, determines the functioning of the lens of the eye.

The method of measuring refractive index - refractometry - is widely used in the practice of laboratory studies to determine the concentration of a substance in a solution, establishing its authenticity and purity.

1. Theory:

Let two transparent media be in contact, and a beam of light from the first medium to the second (Fig. 1, a, b). Then, reflection and refraction (refraction) of light takes place at the interface between the two media.

Absolute refractive index of the medium

$$n = \frac{c}{\upsilon}, \tag{1}$$

where c- is the speed of light propagation in vacuum; υ-is the speed of light propagation in a given medium.

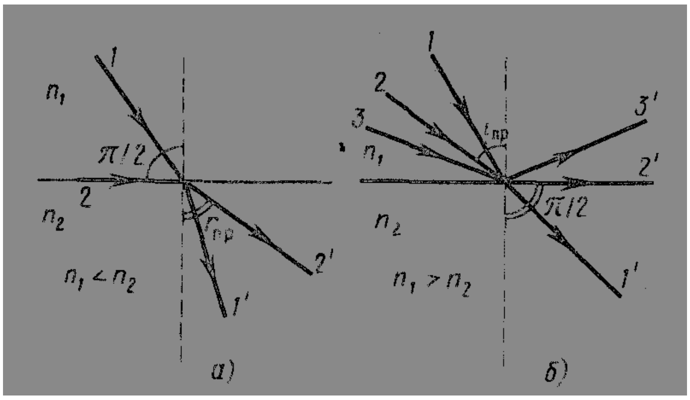


Fig. 1

Relative refractive index of media

$$n_{21} = \frac{n_2}{n_1} , \tag{2}$$

where n_2 and n_1 are the absolute refractive indices of the media.

When light passes from a medium with a smaller refractive index (an optically less dense medium) to a medium with a large refractive index (an optically denser medium), the angle of incidence of the beam is greater than the angle of refraction (Fig. 1, a). If the beam falls on the interface of media at the largest possible angle $i = \frac{\pi}{2}$ (the beam slides along the interface of media), then it will be refracted at an angle $r_{\lim} < \frac{\pi}{2}$. This angle is the largest refraction angle for these media and is called the limiting refraction angle. From the law of refraction of light follows

$$n_{21} = \frac{\sin\left(\frac{\pi}{2}\right)}{\sin r_{\lim}} = \frac{1}{\sin r_{\lim}} = \frac{n_2}{n_1}, \tag{3}$$

whence

$$\sin r_{\lim} = \frac{n_1}{n_2}, \qquad (4)$$

If light passes from optically denser medium to optically less dense, then the angle of refraction is greater than the angle of incidence (Fig. 1, b). At a certain angle i of incidence of the ray, the angle of refraction is equal $\frac{\pi}{2}$, i.e. The refracted beam slides along the media interface. With further increase in the angle of incidence, refraction does not occur; all incident light is reflected from the media interface (full reflection). The angle i is called the limiting angle of total reflection and is denoted $i_{\lim}$. As

$$n_{21} = \frac{\sin i_{\lim}}{\sin\left(\frac{\pi}{2}\right)} = \frac{n_2}{n_1}, \qquad (5)$$

then

$$\sin i_{\lim} = \frac{n_2}{n_1}, \qquad (6)$$

Thus, the limiting angle of refraction and the limiting angle of total reflection for these media depend on their refractive indices. This has found application in devices for measuring the refractive index of substances - refractometers (Fig. 2, a, b, c) used in determining the purity of water, the concentration of the total protein of blood serum, to identify various substances, etc.
The refractive index of solutions is determined by the refractive index of the solvent and depends linearly on the concentration of the solute:

$$n = n_0 + Ac, \qquad (7)$$

where A- is the proportionality coefficient characteristic of a given solute. The values of this coefficient are determined with high accuracy for many substances. Therefore, formula (1) can be used to measure the concentration of various substances using refractive index measuring instruments-refractometers.

The refractometer is designed to measure the refractive index of solutions of various substances. The principle of the refractometer is to measure the limiting angle of refraction at the boundary of the liquid and the glass prism with a known refractive index.

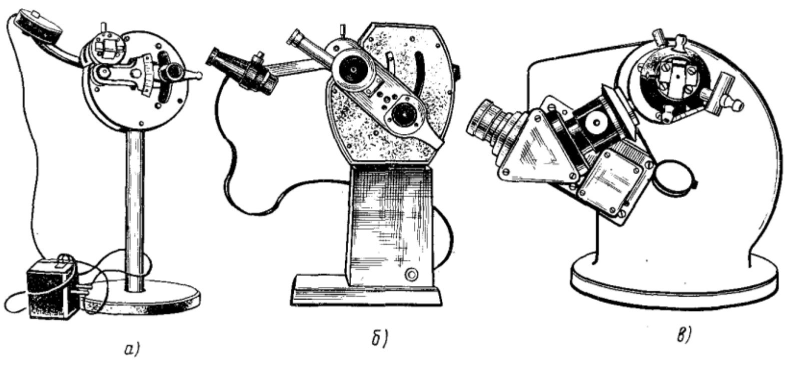


Fig. 2

The refractometer consists of two prisms: an auxiliary folding prism 1 with a matt lower edge 2 and a measuring prism 3. The prisms touch the hypotenuse faces between which there is a gap of about 0.1 mm into which a few drops of the liquid are placed 4. The limiting refraction angle at the boundary liquid is a measuring prism.

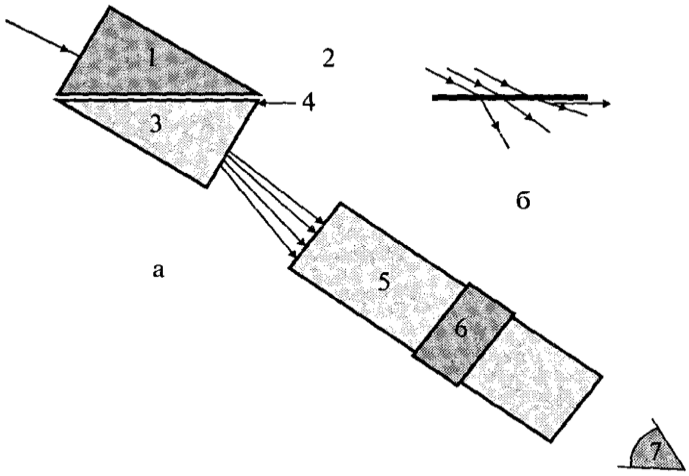


Fig.3 - block diagram of the refractometer: 1 - auxiliary folding prism with a matt face 2; 3 - measuring prism; 4 - the investigated liquid; 5 - reading device; 6 - the compensator; 7 - the eye; b - scheme of light scattering by the matte bottom face 2 of the folding prism.

The magnitude of the angle and, accordingly, the refractive index is determined with the help of the reading device 5. The compensator 6 built into it makes it possible to make the light-shadow boundary black and white when illuminated with white light.

A ray of light passing through the auxiliary folding prism 1 is scattered on the matte lower face 2. The scattered rays propagate in all directions, including parallel to the surface of the measuring prism 3 (Fig. 3b).

Further, these rays are refracted at the boundary of the liquid 4-measuring prism 3, and, passing through this prism 3, fall into the reading device 5. If the refractive index of the liquid is less than the refractive index of the glass, then the rays of light enter the prism 3 in the range from 0 to $r_{\lim}$. The space inside this corner will be illuminated, and outside it - dark. Thus, the field of view visible in the telescope is divided into two parts: dark and light. The position of the interface between light and shadow is determined by the limiting refraction angle, which depends on the refractive index of the liquid being studied.

If the liquid under investigation has a large absorption index (turbid, colored liquid), then in order to avoid energy loss when light passes through the liquid, the measurements are carried out in reflected light. If the liquid is optically less dense than the glass from which the prism is made, then the rays falling at angles larger $i_{\lim}$ will experience total internal reflection and exit through the second side face of the measuring prism into the telescope. The field of vision, as in the first case, will be divided into light and dark parts. The position of the interface in this case is determined by the limiting angle of total reflection $i_{\lim}$, which also depends on the refractive index of the liquid being studied..

If the light-shadow boundary turns out to be colored and blurred, then using the compensator 6, one must achieve a sharp black-and-white border. The construction of the reading device 5 is such that, by means of a special lever, it is possible to combine the light-shadow boundary with the marker of the reading device. In this case, the marker shows directly on the built-in scale the refractive index values.

PERFORMANCE ORDER

The design of the RPL-3 refractometer is shown schematically in Fig.4

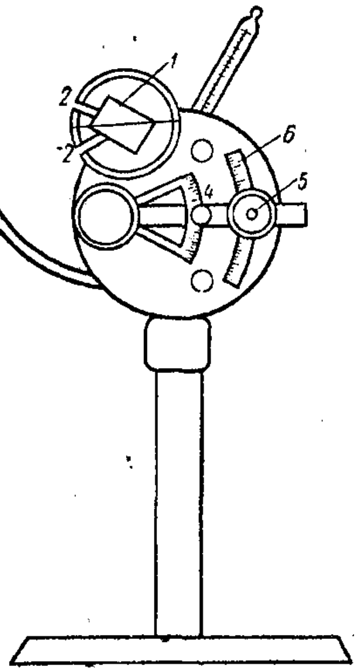


Fig.4

The device consists of a body, which has the form of a flat round box, screwed onto a tripod with a massive base. The main part of the refractometer is prism 1, which consists, in fact, of two rectangular prisms folded by hypotenuse and mounted in hollow casings, made in the form of half-cylinders. The bottom of them is fixed to the body, and the upper one is pivoted on the hinge.

Both prisms are made of heavy glass with a refractive index of the order of 1.7 and are mounted in semi-cylinders so that when the latter are folded, a free space of about 0.15 mm remains between the prisms facing each other and parallel to each other. This space is filled during the measurement with the test liquid. On the left side of each half cylinder there is an opening 2 (Fig. 4) through which the light reflected from the mirror 3 can be directed to the prism 1. The light beam passing through the prism 1 meets in its path a rectangular rotating prism that changes the direction of the beam by 900 , and directs it to the telescope 5.

The determination of the refractive index of a liquid using a refractometer, as mentioned above, can be performed in two ways (in transmitted and reflected light). Let us consider them in more detail.

In the first method - in transmitted light (Fig. 5, a) - a beam of light rays emitted by a light source S, with the help of a mirror Z is directed to the face AB of the ABC prism. Breaking on the face of AB, the rays pass into the prism of ABC and reach the face of the AU. But since this face is made of matte and therefore causes the scattering of light, the rays enter the liquid and reach the edge of the DE at different angles. It is obvious that the greatest possible angle of incidence for rays incident on the face of DE is 90 °. These rays, sliding along the surface DE, after refraction determine the boundary of the propagation of light, since they correspond to the limiting angle of refraction.

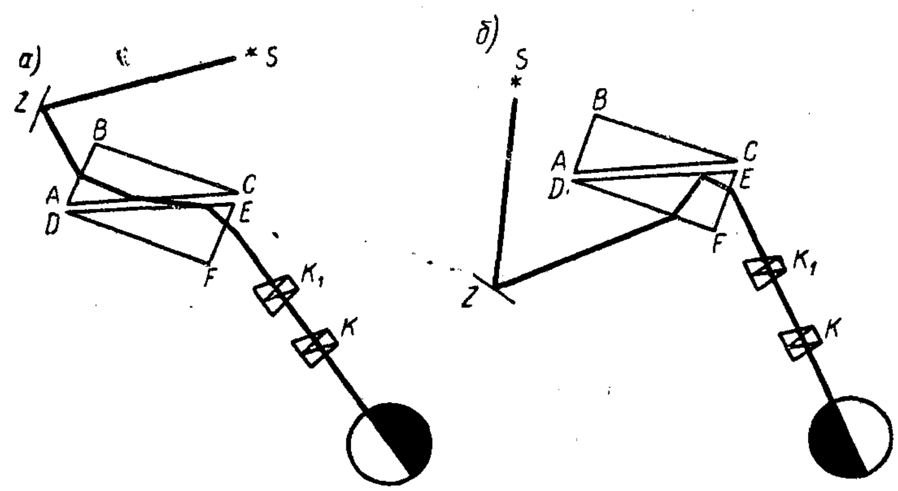


Fig. 5

In the second method, in reflected light (Fig. 5, b), a beam of light rays emitted by the source S is directed to the face DF by means of a mirror Z. Since the face of DF is also matte, the rays enter the DEF prism at different angles. In this case, the rays entering the DEF prism and reaching the DE face must move from the optically denser medium (glass) to the optically less dense medium (liquid). The rays incident on the DE surface at an angle less than the limiting angle will pass into the liquid and into the prism of the ABC. The rays, in which the angle of incidence is greater than the limit, will undergo a complete internal reflection. The rays, whose direction corresponds to the magnitude of the limiting angle, determine the interface between light and shadow.

In the case of colorless and slightly colored liquids, it is convenient to use the first method. When measuring the refractive index of intensely colored liquids that strongly absorb light, it is better to use the second method.

With the correct position of the compensator in the eyepiece of the telescope visible field of view divided into two parts with a sharp border light - shadow, without color shades. Simultaneously, a scale 6 (see Fig. 4) is visible in the eyepiece, on which the refractive indices and the sight line (three dashes) are plotted on the left. The sight line (marker) is applied to the glass located inside the telescope. During measurements, the eyepiece of the telescope moves along the scale until the sight line coincides with the light-shadow interface. At the left, on the scale, you can directly read the value of the refractive index n.

Preparing the device for operation

1. Fold the top prism of the refractometer and pipetting 2-3 drops of distilled water onto the lower prism. Lower the top prism;
2. Orient the refractometer so that observations are made in transmitted light;
3. Focusing the eyepiece of the telescope 5 (Fig. 4) and rotating the mirror, achieve bright illumination of the field of view;
4. Slowly move the eyepiece up along the scale until a light-shadow appears in the field of view. The section line should be sharp and without color. The latter is achieved by turning the handle of the compensator (lever 4 (see Fig. 4));
5. Moving the eyepiece, combine the sight line (three strokes) with the light-shadow interface. Calculate the value of the refractive index of the liquid on the left scale. With the correct setting of the refractometer, the scale reading should correspond to the refractive index of water n = 1,333-1,334.

The task. The dependence of the refractive index of the NaCl solution on the concentration

1. For three solutions of salt with known concentrations, perform three measurements of the refractive index n', n'', n''' .,,. To do this, apply to the lower prism alternately solutions of different concentrations and, combining the viewer with the light-shadow interface, determine the refractive index of the solutions on the scale. For each solution, measure the refractive index three times and calculate the average value of the refractive index n;
2. Record the measurement results in the table;
3. Construct a calibration line: the graph of the dependence of n on the concentration c;
4. Measure the refractive index n_x of a solution of unknown concentration;
5. Determine the concentration c_x of this solution from the graph and place its value in the table.

Table

Substance	Concentration c, g/l	refractive index			
		n'	n''	n'''	n
water	0				
NaCl	c_1				
	c_2				
	c_3				
	c_x				

3. The purpose of the activity of students in the class:

The student should know:

1. Laws of refraction and reflection of light.
2. The path of the rays in the transition of light from optically denser to a less dense medium.
3. The course of the rays in the transition of light from optically less dense to a denser medium.
4. The physical meaning of the absolute and relative refractive indices.
5. Purpose, principle of operation and device refractometer.

The student should be able to:

1. Measure the refractive index of a solution using a refractometer.
2. Investigate the dependence of the refractive index of the solution on the concentration.

4. Training content:

1. Laws of reflection and refraction of light.
2. Purpose of the refractometer.
3. The device and the principle of the refractometer.
4. Measure the refractive index and determine the concentration of the solution.

5. List of questions to check the initial level of knowledge:

1. Formulate the laws of reflection and refraction of light.
2. What is the limiting angle of refraction?
3. What is the phenomenon of total internal reflection?
4. What is the limiting angle of total internal reflection?

6. List of questions for verifying the final level of knowledge:

1.Explain the purpose, principle of operation and the device of the refractometer.

2. Describe methods for determining the refractive index of liquids in transmitted and reflected light, draw the ray path in a refractometer in these cases.

7.Chronocard academic lesson:

1. Organizational moment - 5 min.
2. Current knowledge control - 40 min.
3. Explanation of the work-10 min.
4. Performance and execution of work - 60 min.
5. Check work and homework - 20 min.

8. List of educational literature for the lesson:

1. L.V. Kukharenko, O.V. Nedzved, M.V. Goltsev, V.G. Leshchenko, “Medical and biological physics for medical students”, Minsk BSMU 2016.